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Hidden attractor in smooth Chua systems

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Abstract. The hidden oscillations (*a basin of attraction of which does not contain neighborhoods of equilibria*) have been obtained first in the 50-60s of 20th century in automatic control systems with scalar piecewise-linear nonlinearity. This brings up the question about the excitation nature of hidden oscillations.

In the present paper it is shown that hidden oscillations can exist not only in systems with piecewise-linear nonlinearity but also in smooth systems. Here it is demonstrated the possibility of the existence of hidden chaotic attractor in modified Chua's system with a smooth characteristic of nonlinear element.

Keywords: chaotic hidden attractor, smooth Chua system, Chua circuits, hidden oscillation, describing function method

1 Introduction

In the initial period of development of the theory of nonlinear oscillations in the first half of last century, a main attention has been given to analysis and synthesis of oscillating systems. For these systems the solution of existence problem of oscillating regimes did not encounter severe difficulties.

The development of modern computers allows one to perform numerical simulation of complex nonlinear dynamical systems and to obtain new information about the structure of their trajectories. In well-known systems of Duffing [1], Van der Pol [2], Beluosov-Zhabotinsky [3], Lorenz [4], Rössler [5] and many others the classical self-exited oscillations and attractors can be studied, with relative ease, numerically by *standard computational procedure, in which after transient process a trajectory, starting from a point of unstable manifold in a small neighborhood of unstable equilibrium, reaches an attractor and computes it.*.

However, the possibilities of this approach turned out to be very limited. In the middle of last century in the systems with scalar nonlinearity there were obtained the oscillations of another type, namely *hidden oscillations, a basin of attraction of which does not contain neighborhoods of equilibria* and which cannot be computed with the help of the above standard procedure. In addition, in this case the integration of trajectories with random initial data is unlikely to furnish the desired result (see, e.g., Arnold's description of Kolmogorov's experiment on the search of limit cycles [6, 8, 7]) since a basin of attraction can be very small and the attractor dimension itself can be much less than the dimension of the considered system. Note, that similar difficulties arise in computation of rare attractors [9].

In 1961 Gubar' [10] showed analytically the possibility of existence of hidden oscillation in two-dimensional dynamical system of phase locked-loop with piecewise-constant impulse nonlinearity. In 50-60's of last century the investigations of widely known Markus-Yamabe's [11], Aizerman's [12], and Kalman's [13] conjectures on absolute stability, have led to the finding of hidden oscillations in dynamical model of automatic control systems with scalar piecewise-linear nonlinearity, which belongs to the sector of linear stability (see [14, 15, 17, 18] and others). Note that in such systems for the investigation of the existence of hidden oscillations, the property of piecewise-linearity of nonlinearity allows one to integrate a system on intervals of linearity and then to apply Andronov's point-transformation method and the estimation of trajectories on Poincare

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cross-section. Note, that similar investigations were carried out also for non-autonomous systems (see, e.g. [19, 19]).

Recently, in 2010, for the first time, a chaotic hidden oscillations (hidden attractors) were discovered in Chuas circuit [21, 22, 18] in Chua's circuits with continuous piecewise-linear nonlinearity. Note that L. Chua himself, analyzing in the work [23] different cases of attractor existence in Chua's circuit, does not expect the existence of such hidden attractor in his system.

Below it will be shown that the existence of hidden oscillations does not relate directly, however, with the property of piecewise-linearity and requires further development of effective methods for their investigation.

In addition, on the example of modified Chua's system with smooth nonlinearity it is shown the existence of chaotic hidden oscillations (hidden attractors) in smooth systems. For localization of hidden attractors an effective analytical-numerical algorithm is suggested.

2 Analytical-numerical method for hidden oscillations localization

Consider a system with vector nonlinearity

$$\frac{d\mathbf{x}}{dt} = \mathbf{P}\mathbf{x} + \boldsymbol{\psi}(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^n. \tag{1}$$

Here $\mathbf{P}$ is a constant $(n \times n)$-matrix, $\boldsymbol{\psi}(\mathbf{x})$ is a continuous[3] vector-function, and $\boldsymbol{\psi}(0) = 0$.

Define a matrix $\mathbf{K}$ in such a way that the matrix

$$\mathbf{P}_0 = \mathbf{P} + \mathbf{K} \tag{2}$$

has a pair of purely imaginary eigenvalues $\pm i\omega_0$, $(\omega_0 > 0)$ and the rest of its eigenvalues have negative real parts. Rewrite system (1) as

$$\frac{d\mathbf{x}}{dt} = \mathbf{P}_0\mathbf{x} + \boldsymbol{\varphi}(\mathbf{x}), \tag{3}$$

where $\boldsymbol{\varphi}(\mathbf{x}) = \boldsymbol{\psi}(\mathbf{x}) - \mathbf{K}\mathbf{x}$.

Introduce a finite sequence of functions $\boldsymbol{\varphi}^0(\mathbf{x})$, $\boldsymbol{\varphi}^1(\mathbf{x}), \ldots, \boldsymbol{\varphi}^m(\mathbf{x})$ such that the graphs of neighboring functions $\boldsymbol{\varphi}^j(\mathbf{x})$ and $\boldsymbol{\varphi}^{j+1}(\mathbf{x})$ slightly differ from one another, the function $\boldsymbol{\varphi}^0(\mathbf{x})$ is small, and $\boldsymbol{\varphi}^m(\mathbf{x}) = \boldsymbol{\varphi}(\mathbf{x})$. Taking into account a smallness of the function $\varphi^0(\mathbf{x})$, one can use and justify mathematically strictly [25, 26, 8, 24] the method of harmonic linearization (the describing function method) for the system

$$\frac{d\mathbf{x}}{dt} = \mathbf{P}_0\mathbf{x} + \boldsymbol{\varphi}^0(\mathbf{x}) \tag{4}$$

and determine a stable nontrivial periodic solution $\mathbf{x}^0(t)$.

For the localization of attractor of original system (3), we shall follow numerically the transformation of this periodic solution with increasing j. This periodic solution, denoted further by $\mathcal{A}_0$, is regarded as a starting *oscillating attractor (an attractor, not including equilibria).*

Here two cases are possible. In the first case all the points of $\mathcal{A}_0$ are in the attraction domain of attractor $\mathcal{A}_1$, which is an oscillating attractor of the system

$$\frac{d\mathbf{x}}{dt} = \mathbf{P}_0\mathbf{x} + \boldsymbol{\varphi}^j(\mathbf{x}) \tag{5}$$

[3]This condition can be weakened if a piecewise-continuous function being Lipschitz on closed continuity intervals is considered [8]

with $j = 1$. In the second case, in the change from system (4) to system (5) with $j = 1$, it is observed a loss of stability (bifurcation) and the vanishing of $\mathcal{A}_0$. In the second case the solution $\mathbf{x}^1(t)$ can be determined numerically by starting a trajectory of system (5) with $j = 1$ from the initial point $\mathbf{x}^0(0)$. If in the process of computation the solution $\mathbf{x}^1(t)$ is not fallen to equilibrium and is not increased indefinitely (here it should be considered a sufficiently large computational interval $[0, T]$), then this solution reaches attractor $\mathcal{A}_1$. Now it is possible to consider system (5) with $j = 2$, to perform a similar procedure of computation of $\mathcal{A}_2$ by starting a trajectory of system (5) with $j = 2$ from the initial point $\mathbf{x}^1(T)$ and to compute the trajectory $\mathbf{x}^2(t)$.

Proceeding then this procedure, sequentially increasing j, and computing $\mathbf{x}^j(t)$ (which is a trajectory of system (5) with initial data $\mathbf{x}^{j-1}(T)$) we either arrive at the computation of $\mathcal{A}_m$ (which is an attractor of system (5) with $j = m$, i.e. original system (3)), either, at a certain step, observe a loss of stability (bifurcation) and the vanishing of attractor.

Determine the initial data $\mathbf{x}^0(0)$ of starting periodic solution $\mathbf{x}^0(t)$. For this purpose, by the linear nonsingular transformation $\mathbf{S}$, system (4) with nonlinearity $\boldsymbol{\varphi}^0(\mathbf{x})$ is reduced to the form

$$\begin{aligned} \dot{y}_1 &= -\omega_0 x_2 + \varepsilon \varphi_1(y_1, y_2, \mathbf{y}_3), \\ \dot{y}_2 &= \omega_0 x_1 + \varepsilon \varphi_2(y_1, y_2, \mathbf{y}_3), \\ \dot{\mathbf{y}}_3 &= \mathbf{A}\mathbf{x}_3 + \varepsilon \boldsymbol{\varphi}_3(y_1, y_2, \mathbf{y}_3). \end{aligned} \tag{6}$$

Here y_1, y_2 are scalar values, $\mathbf{y}_3$ is $(n-2)$-dimensional vector; $\boldsymbol{\varphi}_3$ is $(n-2)$-dimensional vector-function, φ_1, φ_2 are scalar functions; $\mathbf{A}_3$ is $((n-2)\times(n-2))$-matrix, all eigenvalues of which have negative real parts. Without loss of generality, it can be assumed that for the matrix $\mathbf{A}_3$ there exists a positive number $d > 0$ such that

$$\mathbf{y}_3^*(\mathbf{A}_3 + \mathbf{A}_3^*)\mathbf{y}_3 \le -2d|\mathbf{y}_3|^2, \quad \forall\, \mathbf{y}_3 \in \mathbb{R}^{n-2}. \tag{7}$$

Introduce the describing function

$$\begin{aligned} \Phi(a) = \int_0^{2\pi/\omega_0} \Big[& \varphi_1\left((\cos\omega_0 t)a, (\sin\omega_0 t)a, 0\right)\cos\omega_0 t + \\ & +\varphi_2\left((\cos\omega_0 t)a, (\sin\omega_0 t)a, 0\right)\sin\omega_0 t \Big]\, dt, \end{aligned}$$

and suppose that for the vector-function $\boldsymbol{\varphi}(\mathbf{x})$ the estimate

$$|\boldsymbol{\varphi}(\mathbf{x}') - \boldsymbol{\varphi}(\mathbf{x}'')| \le L|\mathbf{x}' - \mathbf{x}''|, \quad \forall\, \mathbf{x}', \mathbf{x}'' \in \mathbb{R}^n \tag{8}$$

is satisfied.

Theorem 1 *[24] If a positive a_0 can be found such that*

$$\Phi(a_0) = 0, \quad \left.\frac{d\Phi(a)}{da}\right|_{a=a_0} \ne 0 \tag{9}$$

then for sufficiently small ε there exists periodic solution $\mathbf{x}^0(t)$ with the initial data $\mathbf{x}^0(0) = \mathbf{S}(y_1(0), y_2(0), \mathbf{y}_3(0))^*$, where

$$y_1(0) = a_0 + O(\varepsilon),\ \ y_2(0) = 0,\ \ \mathbf{y}_3(0) = \mathbf{O_{n-2}}(\varepsilon), \tag{10}$$

and $\mathbf{O_{n-2}}(\varepsilon)$ is $(n-2)$-dimensional vector such that all its components are $O(\varepsilon)$.

If the stability is regarded in the sense that for all solutions with the initial data sufficiently close to $\mathbf{x}^0(0)$, the modulus of their difference with $\mathbf{x}^0(t)$ is uniformly bounded for all $t > 0$), then for the stability of $\mathbf{x}^0(t)$ it is sufficient that the condition $\left.\frac{d\Phi(a)}{da}\right|_{a=a_0} < 0$ is satisfied.

Theorem 1 permits one to construct analytical-numerical algorithms for the search of hidden oscillations in various systems with scalar and vector nonlinearities.

3 Localization of hidden attractor in Chua's system with smooth scalar nonlinearity

Classical Chua's attractors are those which are excited from unstable equilibria. This permits one, with relative easy, to compute different classical Chua's attractors [27, 28, 29, 30, 31, 32]. But recently by special analytical-numerical algorithm [24] in classical Chua's circuit there were also discovered the hidden attractors [22].

The above theorem can be used for the search of hidden oscillations in modified Chua's system with smooth scalar nonlinearity.

Consider the following smooth Chua's system

$$\begin{aligned} \dot{x} &= \alpha(y - x) - \alpha f(x), \\ \dot{y} &= x - y + z, \\ \dot{z} &= -(\beta y + \gamma z). \end{aligned} \tag{11}$$

Here the function

$$\begin{aligned} f(x) &= m_1 x + (m_0 - m_1)\tanh(x) = \\ &= m_1 x + (m_0 - m_1)\frac{e^x - e^{-x}}{e^x + e^{-x}} \end{aligned} \tag{12}$$

characterizes a nonlinear element of the system (here it is considered smooth nonlinearity $\tanh(x)$ close to the nonlinearity saturation(x) in the classical Chua's circuit); $\alpha, \beta, \gamma, m_0, m_1$ are parameters of system.

Let us apply the above analytical-numerical algorithm to analysis of Chua's system. For this purpose, rewrite Chua's system (11) as (1)

$$\frac{d\mathbf{x}}{dt} = \mathbf{P}\mathbf{x} + \mathbf{q}\psi(\mathbf{r}^*\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^3. \tag{13}$$

Here

$$\mathbf{P} = \begin{pmatrix} -\alpha(m_1+1) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & -\gamma \end{pmatrix}, \ \mathbf{q} = \begin{pmatrix} -\alpha \\ 0 \\ 0 \end{pmatrix}, \ \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$
$$\psi(\sigma) = (m_0 - m_1)\tanh(\sigma).$$

Introduce the coefficient k and small parameter ε, and represent system (13) as (4):

$$\frac{d\mathbf{x}}{dt} = \mathbf{P_0}\mathbf{x} + \mathbf{q}\varepsilon\varphi(\mathbf{r}^*\mathbf{x}). \tag{14}$$

Here

$$\mathbf{P_0} = \mathbf{P} + k\mathbf{q}\mathbf{r}^* = \begin{pmatrix} -\alpha(m_1+1+k) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & -\gamma \end{pmatrix},$$
$$\lambda_{1,2}^{\mathbf{P}_0} = \pm i\omega_0, \ \lambda_3^{\mathbf{P}_0} = -d,$$
$$\varphi(\sigma) = \psi(\sigma) - k\sigma = (m_0 - m_1)\tanh(\sigma) - k\sigma.$$

Consider system (14) with the parameters

$$\begin{aligned} &\alpha = 8.4562, \ \beta = 12.0732, \ \gamma = 0.0052, \\ &m_0 = 0.3532, \ m_1 = -1.1468. \end{aligned} \tag{15}$$

Note that for the considered values of parameters, in the system there are three equilibria: one locally stable zero equilibrium and two saddle equilibria.

Now we apply the above procedure of hidden attractors localization to Chua's system (13) with parameters (15). For this purpose a starting frequency and a coefficient of harmonic linearization are computed. We have

$$\omega_0 = 2.0392, \quad k = 0.2098\,.$$

Compute then the solutions of system (14) with nonlinearity $\varepsilon\varphi(x) = \varepsilon(\psi(x) - kx)$, sequentially increasing ε from the value $\varepsilon_1 = 0.1$ to $\varepsilon_{10} = 1$ with the step 0.1.

By (10) and the matrix $\mathbf{S}$ (which can be found using [22]) for the transformation of system (14) to the form (6), one can obtain the initial data, namely

$$x(0) = 8.8200, y(0) = 0.5561, z(0) = -12.6008$$

for the first step of multistage procedure for localization of hidden oscillation. For the value of parameter $\varepsilon_1 = 0.1$, after transient process the computational procedure reaches the starting oscillation $\mathbf{x}^1(t)$ (Fig. 1). Then by sequential increasing the parameter ε_j and the computation of $\mathbf{x}^j(t)$ (see (Figs. 1-**??**)), the set $\mathcal{A}_{\text{hidden}}$ is computed for original Chua's system (13).

The set $\mathcal{A}_{\text{hidden}}$ is shown in Fig. 3.

It should be noted that the decreasing of integration step, the increasing of integration time, and the computation of different trajectories of original system with initial data from a small neighborhood of $\mathcal{A}_{\text{hidden}}$ lead to the localization of the same set $\mathcal{A}_{\text{hidden}}$ (all the computed trajectories densely trace the set $\mathcal{A}_{\text{hidden}}$). Note also that for the computed trajectories it is observed Zhukovsky instability and the positiveness of Lyapunov exponent [33, 34][4].

The behavior of system trajectories in the neighborhood of equilibria is shown in Fig. 3, where $M^{\text{unst}}_{1,2}$ are unstable manifolds and $M^{\text{st}}_{1,2}$ are stable manifolds. It follows that in a phase space of system there are stable separating manifolds of saddles.

Thus, by the above and, taking into account the remark on the existence, in system, of locally stable zero equilibrium F_0, which attracts the stable manifolds $M^{\text{st}}_{1,2}$ of two symmetric saddles S_1 and S_2, we arrive at the conclusion that in $\mathcal{A}_{\text{hidden}}$ a hidden strange attractor is computed.

[4] Lyapunov exponents (LEs) were introduced by Lyapunov for the analysis of stability by the first approximation for *regular* time-varying linearizations, where negativeness of the largest Lyapunov exponent indicated stability. Later Chetaev proved that for *regular* time-varying linearizations positive Lyapunov exponent indicated instability (a gap in his work is discussed and filled in [34]). While there is no general methods for checking regularity of linearization and there are known Perron effects [34] of the largest Lyapunov exponent sign inversions for non regular time-varying linearizations, computation of Lyapunov exponents for linearization of nonlinear autonomous system along non stationary trajectories is widely used for investigation of chaos, where positiveness of the largest Lyapunov exponent is often considered as indication of chaotic behavior in considered nonlinear system.

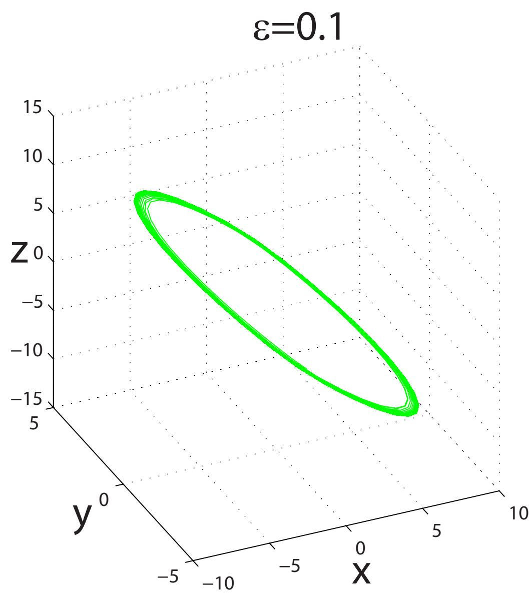

Figure 1: Oscillation localization.

Figure 2: Oscillation localization.

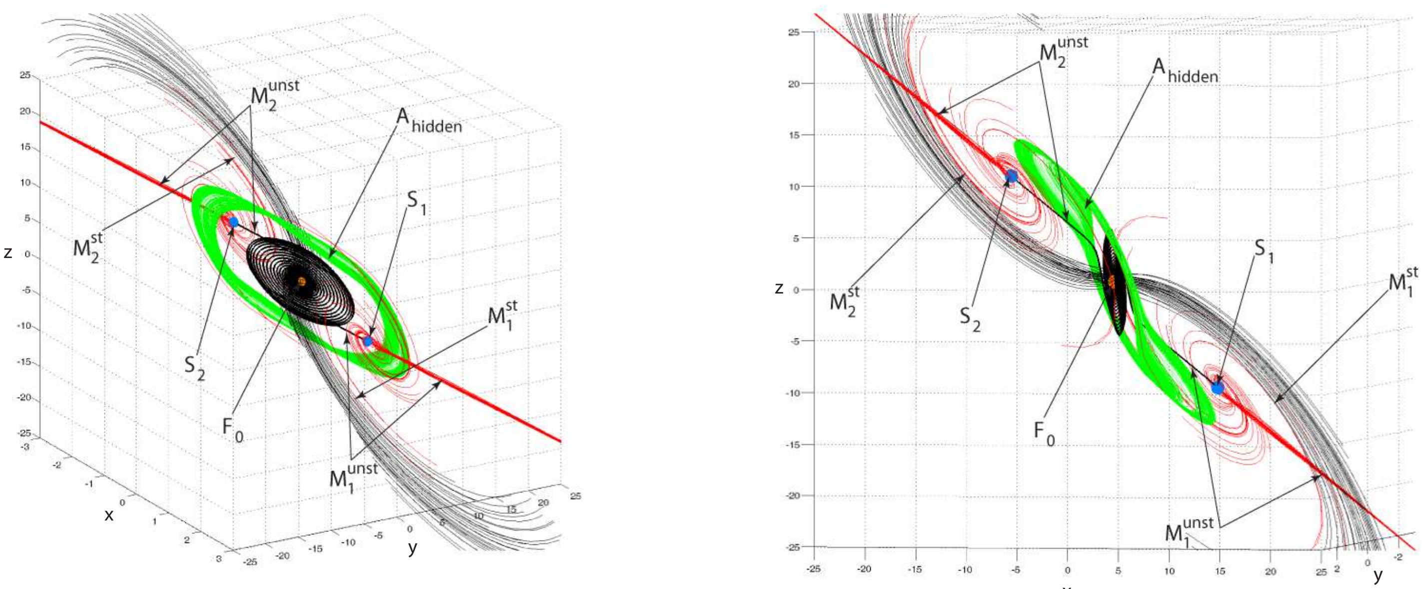

Figure 3: Equilibrium, stable manifolds of saddles, and localization of hidden attractor.

4 Conclusions

In the present paper the application of special analytical-numerical algorithm to localization of hidden attractor is discussed and the existence of such hidden attractor in smooth Chua's system is demonstrated.

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