# Book Review

**Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities** — V.A. Yakubovich, G.A. Leonov, and A.Kh. Gelig (New Jersey: World Scientific, 2004).
*Reviewed by Valery Yurkevich and Ardeshir Guran*

## I. Introduction

Many nonlinear systems of practical importance can be represented in the form of a serial connection between a linear dynamical system and an algebraic block of nonlinear functions. Two problems are often treated for such systems: (1) the existence and stability of periodic solutions, and (2) the absolute stability of the given nonlinear feedback configuration.

The describing function method is a powerful tool for investigating the existence and stability of periodic solutions in systems having nonsmooth nonlinearities, e.g., those associated with saturation, relay, dead zone, or hysteresis [1], [4], [2], [8].

The present book is aimed at the second problem mentioned above: that is, the stability analysis of that particular type of nonlinear system, based on the Lyapunov function approach together with frequency-domain methods.

The book centers on Lure's problem [7], i.e., the conditions for absolute stability of a feedback connection between a linear system and a nonlinear algebraic block with given sector conditions. Aizerman's conjecture [3] is well known. Suppose that a linear system, where a nonlinear algebraic block has been replaced by gain, is stable for all values of gain from some given interval. Then this linear system with a nonlinear algebraic block, where the sector conditions are defined by the same gain interval, is globally uniformly asymptotically stable at the origin. Unfortunately, the conjecture does not hold in general [9]. For the purpose of absolute stability analysis, such sufficient frequency-domain conditions as the circle and Popov criteria were developed [10], [11], [5], [12], [13].

Although more than a thousand publications are devoted to the problem of absolute stability analysis [6], investigation in this area is still growing rapidly. The book by Yakubovich, Leonov, and Gelig provides an example of such investigation, in which the methods of absolute stability theory are used for the analysis of systems governed by nonlinear differential equations with discontinuous right-hand sides. The subject matter is important from both theoretical and practical viewpoints.

## II. The book

A distinctive feature of the nonlinear systems treated in the book is the presence of nonunique equilibria. The investigation is based on a Lyapunov function approach, with functions constructed through the solution of matrix inequalities. Solvability of these inequalities is related to frequency-domain stability criteria.

The main goals of the book are as follows: (1) the further development of control theory for differential equations with discontinuous nonlinearities, in particular, the frequency-domain criteria for dichotomy (non-oscillation) and for stability of equilibria sets; (2) an elucidation of the dynamics of specific technical systems.

Chapter 1 provides an introduction to the theory of differential equations with discontinuous nonlinearities, in particular, of differential equations having multiple-valued right-hand sides. A nonlinear characteristic of a dry friction serves to illustrate a multiple-valued nonlinearity. A notion of solution for such systems is introduced, and its relationship to other definitions of solution to a system with discontinuous right-hand side is discussed. The distinctive feature of the approach is that dynamical systems having discontinuous right-hand sides are treated as differential inclusions. It is shown that this approach is much more general than other known approaches. The concept of sliding mode is considered as a solution that slides along a discontinuity surface. Local existence, continuation of solutions, and continuous dependence on initial values are emphasized. The chapter concludes with notions of dichotomy, global asymptotic stability, and piecewise asymptotic stability, as well as with Lyapunov-type lemmas for various types of stability.

Chapter 2 explains how the construction of Lyapunov functions, which guarantee global stability of nonlinear systems, leads to some algebraic problems on the solution of matrix inequalities. The frequency-domain conditions on solvability of quadratic matrix inequalities are obtained. Then, the circle and Popov criteria for absolute stability are deduced. Various versions of the Kalman–Yakubovich lemma on solvability of matrix inequalities are presented and discussed in detail. For the sake of completeness, some algebraic statements related to controllability, observability, and stabilizability are presented as well.

Chapter 3 is devoted to dichotomy and stability analysis of nonlinear systems having multiple equilibria. Frequency-domain criteria for dichotomy, quasi-gradient-like behavior, and pointwise global stability are derived. It has been shown that the resulting frequency-domain criteria cannot be improved if the Lyapunov functions are taken from some specified class of functions. The established criteria are used to study the gyroscope, turbine, autopilot, and other technical systems.

Chapter 4 addresses the stability of equilibria sets of pendulum-like systems described by ordinary differential equations with right-hand sides periodic in some coordinates. Conditions for global stability of the stationary set of multidimensional systems are obtained through the method of

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periodical Lyapunov functions. An analogue of the circle criterion for pendulum-like systems is deduced by construction of the net of invariant cones in the system's phase space. The method established by G.A. Leonov is discussed, which allows one to reduce the investigation of multidimensional pendulum-like systems to the study of certain two-dimensional systems together with verification of the frequency-domain criteria. The frequency-domain stability criteria obtained in this chapter are applied to phase synchronization systems.

Finally, advanced proofs are collected in an appendix.

## III. COMMENTS

Throughout the text, there are many interesting comments and pointers to references concerned with the history of the subject. The main results presented were original contributions of the authors. All topics are closely related to the problem of stability analysis of systems having discontinuous nonlinearities, based on frequency-domain methods.

Unfortunately, the book is not intended as a teaching text and some useful pedagogical devices (end-of-chapter summaries, problem sets, etc) are absent. The authors, as originators of the subject matter, could heighten their impact on the control system community by producing (as a second edition or, possibly, a new volume) a text aimed directly at students and further emphasizing practical applications.

The reviewer recommends this book: first, to mathematicians interested in control theory and, second, to researchers interested in systems having nonsmooth nonlinearities and their technical applications.

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