Discontinuous Load Rating Problem for Induction Motors

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The transient processes, arising after discontinuous loading the induction motors, are considered. The estimate of discontinuous load rating is obtained.

1 Introduction

The problem of discontinuous load rating for synchronous machine is considered in (Yanko-Trinitskii, 1958; Leonov, 2001). For its solving some special methods for constructing the Lyapunov functions are developed (Yanko-Trinitskii, 1958; Leonov, 2001; Yakubovich, 2004).

In the present paper the similar problem is considered for induction motors. For its solving the method of nonlocal reduction is proposed (Leonov, Burkin, 1996; Leonov, Ponomarenko, 1996).

The problem of discontinuous load rating for induction motors is considered taking into account the work of turner, which makes a certain detail on the turning machine. (Note that in the machine axis drives the induction motors are used). The stages of this work are the following. At first the turner starts a turning machine and, by that, an induction motor. After a fast transient process a free movement occurs. At the beginning of the process of cutting action the moment of force M arises discontinuously and acts on the rotor of motor. Then a transient process from a free movement to a stationary turning of rotor with the drag torque M occurs. The object of this paper to find the permissible M for which the induction motor changes over from the mode being a free movement to the operating mode of cutting action.

In the present paper it is shown that the load rating are those satisfying the following inequality

$$M \leq \frac{\sqrt{3}}{2}\, M_H^{\max}.$$

Here $M_H^{\max}$ is a maximum electromagnetic torque, which is a maximum of static mechanical characteristic (Kloss characteristic) of induction motor.

The various improvements of this estimate are also obtained.

2 Formulation of the problem

Let us consider the system of differential equations of induction motor

$$\begin{aligned}
&\left(L_s \frac{d}{dt} + R_s\right) i_\alpha^s + \kappa\, L\, \frac{d}{dt}\,(i_\alpha^r \cos\gamma - i_\beta^r \sin\gamma) = -u_m \sin\omega_s t,\\
&\left(L_s \frac{d}{dt} + R_s\right) i_\beta^s + \kappa\, L\, \frac{d}{dt}\,(i_\alpha^r \sin\gamma + i_\beta^r \cos\gamma) = u_m \cos\omega_s t,\\
&L\,\frac{d}{dt}\,(i_\alpha^s \cos\gamma + i_\beta^s \sin\gamma) + \left(L_r \frac{d}{dt} + R_r\right) i_\alpha^r = 0,\\
&L\,\frac{d}{dt}(-i_\alpha^s \sin\gamma + i_\beta^s \cos\gamma) + \left(L_r \frac{d}{dt} + R_r\right) i_\beta^r = 0,\\
&J\ddot{\gamma} = L[(i_\alpha^r i_\beta^s - i_\beta^r i_\alpha^s)\cos\gamma - (i_\alpha^r i_\alpha^s + i_\beta^r i_\beta^s)\sin\gamma] - M_H.
\end{aligned} \tag{1}$$

Here variables $i^s_\alpha, i^s_\beta, i^r_\alpha, i^r_\beta$ are the currents of stator and rotor windings, γ is the angle of turn of the rotor, parameters R_s, L_s, R_r, L_r are resistances and inductances of stator and rotor windings, L is the peak significance of the mutual inductance between them, J is the moment of inertia of the rotor, $\omega_s = 2\pi f, f, u_m$ are the frequency and the amplitude of the voltage brought to the stator windings, $M_H(t)$ is the moment of the load on the shaft of the induction motor, t is the time, κ is the parameter of influence of electromagnetic processes in a rotor on processes in the stator windings. The equations (1) has been considered in (White 1959, Kondrat'eva 2001, Leonov, Kondrat'eva 2001) Neglecting the influence of electromagnetic processes in a rotor on processes in the stator windings we assume in system (1) $\kappa = 0$.

In (Kondrat'eva 2001, Leonov, Kondrat'eva 2001) it is shown that for $\kappa = 0$ system (1) can be reduced to the equations

$$\begin{aligned} \dot{x} &= -\alpha x + sy + 1, \\ \dot{y} &= -\alpha y - sx, \\ \dot{s} &= \delta[\alpha\beta y + M(t)]. \end{aligned} \tag{2}$$

Here x and y are the linear combinations of the currents $i^s_\alpha, i^s_\beta, i^r_\alpha, i^r_\beta$, s is a slip:

$$s = 1 - \dot{\gamma}/\omega_s, \quad \alpha = \frac{R_r}{\omega_s L_r},$$

$$\beta = \frac{L^2 \omega_s^2 L_s}{L_r(R_s^2 + \omega_s^2 L_s^2)},$$

$$\delta = \frac{u_m^2}{\omega_s^4 J L_s},$$

$$M(t) = \frac{M_H(t)}{\omega_s^2 J \delta}.$$

Consider the problem of load rating for the system of equations (2).

After the start of a motor and the end of a transient process a free movement occurs. This mode corresponds to the relation $M(t) = 0$, $t \in [0, T)$ and to the following solution of system (2): $s(t) = y(t) = 0$, $x(t) = \alpha^{-1}$, $t \in [0, T]$.

At time $t = T$ the loading $M(t) = M > 0$, $t \in [T, +\infty)$ occurs.

We are interested in the value of M for which in a new transient process with the initial data $s(T) = y(T) = 0, x(T) = \alpha^{-1}$ the solution of system (2) tends to a new stable equilibrium as $t \to \infty$.

It is well known (Kondrat'eva, 2001; Leonov, Kondrat'eva, 2001) that a necessary condition for the existence of such stable equilibrium is as follows

$$M \leq M_H^{\max} = \frac{\beta}{2}. \tag{3}$$

We now find a sufficient condition for the existence of transient process with the above properties. Suppose that the following inequality holds

$$M < \frac{2\alpha^2}{\delta}. \tag{4}$$

From relation (3) it follows that inequality (4) is satisfied if

$$\frac{4\alpha^2}{\beta} > \delta. \tag{5}$$

This inequality is usually satisfied for induction motors since δ is a small value, which is inversely proportional to the moment of inertia of rotor.

3 Solution of the discontinuous load rating problem

Consider a function

$$\varphi(s) = \delta\left(-\frac{M}{\alpha}\,s^2 + \beta s - \alpha M\right)$$

and the number

$$a = 2\max_{\lambda\in(0,\alpha)}\left[\lambda\left(\alpha-\lambda-\frac{(\delta M)^2}{4\alpha^2(\alpha-\lambda)}\right)\right]^{1/2}.$$

Relation (4) implies that $a > 0$. From the inequality $M < \beta/2$ it follows that the function $\varphi(s)$ has two zeros: $s = s_0 \in (0, \alpha)$ and $s = s_1 > \alpha$.

Theorem. *Let for the solution $\theta(t)$ of the equation of the second order*

$$\ddot\theta + a\dot\theta + \varphi(\theta) = 0 \tag{6}$$

with the initial data $\theta(0) = \dot\theta(0) = 0$ the following condition holds

$$\theta(t) \le s_1, \quad \forall\, t \ge 0. \tag{7}$$

Then the solution of system (2) *with the initial data $s(T) = y(T) = 0$, $x(T) = \alpha^{-1}$ satisfies the following relations*

$$\begin{aligned}
&\lim_{t\to+\infty} x(t) = \tfrac{M}{\beta s_0}\\
&\lim_{t\to+\infty} y(t) = -\tfrac{M}{\alpha\beta},\\
&\lim_{t\to+\infty} s(t) = s_0.
\end{aligned} \tag{8}$$

Proof. Having performed the change of variables in system (2)

$$\eta = \delta\alpha\beta y + M\delta, \quad z = -\alpha x - \frac{M}{\alpha\beta}\,s + 1,$$

we get

$$\begin{aligned}
&\dot s = \eta,\\
&\dot\eta = -\alpha\eta + \delta\beta s z - \varphi(s),\\
&\dot z = -\alpha z - \tfrac{1}{\delta\beta}\,s\eta - \tfrac{M}{\alpha\beta}\,\eta.
\end{aligned} \tag{9}$$

Equation (6) is equivalent to the equation of the first order, namely

$$F\,\frac{dF}{d\theta} = -aF - \varphi(\theta). \tag{10}$$

Relation (7) implies that there exists the solution $F(\theta)$ of equation (10), defined on the segment $[s_2, s_1]$, $F(s_2) = F(s_1) = 0$, $F(\theta) > 0$, $\forall\,\theta \in (s_2, s_1)$, $s_2 < 0$.

Consider a function

$$V(s,\eta,z) = \frac{(\delta\beta)^2}{2}\,z^2 + \frac12\,\eta^2 - \frac12\,F(s)^2, \quad s \in [s_2, s_1].$$

On the solutions of system (9) $s(t) \in [s_2, s_1]$, $\eta(t)$, $z(t)$ we have the following estimate

$$\dot{V}(s(t),\eta(t),z(t)) = -\alpha\eta(t)^2 - \tfrac{\delta^2\beta M}{\alpha}\,\eta(t)z(t) -$$

$$-\alpha(\beta\delta)^2 z(t)^2 - F'(s(t))F(s(t))\eta(t) - \varphi(s(t))\eta(t) =$$

$$= -2\lambda V(s(t),\eta(t),z(t)) - (\alpha-\lambda-\varepsilon)\eta(t)^2 - \tfrac{\delta^2 M\beta}{\alpha}\eta(t)z(t) -$$

$$-(\beta\delta)^2(\alpha-\lambda)z(t)^2 - (F'(s(t))F(s(t)) + \varphi(s(t)))\eta(t) -$$

$$-\varepsilon\eta(t)^2 - \lambda F(s(t))^2 \leq -2\lambda V(s(t),\eta(t),z(t)) +$$

$$+\tfrac{(F'(s(t))F(s(t))+\varphi(s(t)))^2}{4\varepsilon} - \lambda F(s(t))^2 =$$

$$= -2\lambda V(s(t),\eta(t),z(t)) - \tfrac{1}{4\varepsilon}\left(F'(s(t))F(s(t)) + 2\sqrt{\lambda\varepsilon}F(s(t)) + \varphi(s(t))\right)\cdot$$

$$\cdot\left(F'(s(t))F(s(t)) - 2\sqrt{\lambda\varepsilon}F(s(t)) + \varphi(s(t))\right) =$$

$$= -2\lambda V(s(t),\eta(t),z(t)),$$

where

$$\varepsilon = \alpha - \lambda - \frac{(\delta M)^2}{4\alpha^2}(\alpha-\lambda)^{-1}.$$

Therefore for a certain λ the relation $2\sqrt{\lambda\varepsilon} = a$ is satisfied and we can use relation (10)

$$F'(s(t))F(s(t)) + 2\sqrt{\lambda\varepsilon}F(s(t)) + \varphi(s(t)) = 0.$$

The following estimate, obtained above,

$$\dot{V}(s(t),\eta(t),z(t)) \leq -2\lambda V(s(t),\eta(t),z(t)).$$

and the relations

$$V(s_1,\eta,z) \geq 0, \quad \forall \eta \in R^1, \quad \forall z \in R^1$$

$$V(s_2,\eta,z) \geq 0, \quad \forall \eta \in R^1, \quad \forall z \in R^1$$

imply the positive invariance of the set (Leonov, Burkin 1996)

$$\Omega = \{V(s,\eta,z) < 0, \quad s \in [s_2, s_1]\}.$$

Consider now the following function

$$W(s,\eta,z) = \frac{(\delta\beta)^2}{2}z^2 + \frac{1}{2}\eta^2 + \int\limits_0^s \varphi(\sigma)\,d\sigma.$$

It is evident that

$$\dot{W}(s(t),\eta(t),z(t)) \leq -\lambda(\eta(t)^2 + (\delta\beta)^2 z(t)^2), \tag{11}$$

Taking into account the positive invariance of Ω, estimate (11), and the fact that the point $\eta(T) = z(T) = s(T) = 0$ is in Ω, we find

$$\lim_{t\to+\infty} \eta(t) = 0,$$
$$\lim_{t\to+\infty} z(t) = 0,$$
$$\lim_{t\to+\infty} s(t) = s_0.$$

This is equivalent to relations (8). The proof is completed.

It is well known (Barbashin, 1969) that condition (7) is satisfied if

$$2\int\limits_0^{s_1} \varphi(\sigma)d\sigma + a^2(s_1 - s_0)^2 \geq 0. \tag{12}$$

Relation (12) is satisfied if

$$-\frac{M}{3\alpha}\, s_1^2 + \frac{\beta}{2}\, s_1 - \alpha M \geq 0.$$

This inequality can be written as

$$\frac{2M}{3\alpha}\, s_1 - \frac{\beta}{2} \geq 0. \tag{13}$$

Since $\varphi(s_1) = 0$ and $s_1 > \alpha$, for s_1 we have

$$s_1 = \frac{\alpha(\beta + \sqrt{\beta^2 - 4M^2}\,)}{2M}\,. \tag{14}$$

Relation (14) implies that inequality (13) is satisfied if

$$\frac{\sqrt{3}}{4}\,\beta \geq M. \tag{15}$$

Thus, a permissible load rating is determined by inequality (15). We recall that $M_H^{\max} = \beta/2$. Therefore inequality (15) can be rewritten as

$$M \leq \frac{\sqrt{3}}{2}\, M_H^{\max}. \tag{16}$$

We now apply estimate (12), assuming that δ is very small value as compared with α, β, and M. Put $\lambda = \alpha/2$. Then condition (12) is satisfied if

$$\frac{\alpha^4(\beta^2 - 4M^2)}{M^2} + O(\delta) > 0. \tag{17}$$

In this case in place of estimate (16) we have

$$M < M_H^{\max}, \quad \delta \ll 1.$$

This estimate is in accordance with the asymptotic analysis of differential equations for induction motor in the case that for the small δ they pass to the equation of the first order with Kloss characteristic in right hand side (Leonov, Kondrat'eva, 2001).

Thus, the important problem of discontinuous load rating is considered and the tools for its solving are developed.

References

1. Barbashin, E.A.; Tabueva, V.A.: *Dynamical systems with cylinder phase space.* Nauka, (1969), (in Russian).

2. Kondrat'eva, N.V.; Leonov, G.A.; Rodyukov, F.F.; Shepeljavyi, A.I.: Nonlocal Analysis of Differential Equations of Induction Motors. *Technische Mechanik*, 81, (2001), 75–86.

3. Leonov, G. A.: *Mathematical Problems of Control Theory. An Introduction.* World Scientific, Singapore, (2001).

4. Leonov, G.A.; Burkin, I.M.; Shepeljavyi, A.I.: *Frequency Methods in Oscillation Theory.* Kluwer, Dordrecht, (1996).

5. Leonov, G.A.; Kondrat'eva, N.V.; Rodyukov, F.F.; Shepeljavyi A.I.: Nonlocal Analysis of Differential Equations of Asynchronous Machines. In: *Nonlinear Mechanics*, editors: Matrosov, V.M. and Rumyantsev, V.V. and Karapetyan, A.V. Phismatgiz, (2001), 256–280, (in Russian).

6. Leonov, G.A.; Ponomarenko, D.V.; Smirnova, V.B.: *Frequency Methods for Nonlinear Analysis. Theory and Applications.* World Scientific, Singapore, (1996).

7. White, D.C.; Woodson, H.H.: *Electromechanical Energy Conversation.* John Wiley and Sons, Inc., New York, (1959).

8. Yakubovich, V.A.; Leonov, G.A.; Gelig, A.Ch.: *Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities.* World Scientific, Singapore, (2004).

9. Yanko-Trinitskii, A. A.: *A new method for the analysis of the work of syncronous motor under discontinuous loads.* Gostekhizdat, (1958), (in Russian).

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