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Distributed State Estimation of Moving Targets Using Cyclic Simultaneous Perturbation Stochastic Approximation ⋆

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Abstract: Sensor networks comprised of multiple nodes with sensing, processing and communication capabilities are ubiquitous in tracking systems. However, when a tracking system is required to track a large number of targets, the computation and communication loads arise. One possible solution is to use a distributed scheme. In this paper we propose a distributed multiple target tracking algorithm, which takes into account restrictions on the sensor network functioning. Our method is based on the modification of Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm with a perturbation on the input. For the proposed algorithm we get an upper bound of residual between estimates and show the simulation results.

Keywords: sensor network, distributed estimation, stochastic approximation, target tracking, communication constraints, computational constraints.

1. INTRODUCTION

Technological advancements made it possible to deploy a large number of inexpensive but technically advanced sensors to cover wide areas. Applications in these fields include, for example, intelligent video surveillance at cluttered and crowded places, air traffic control, space situational awareness and animal tracking (see Hanif et al. (2017); Thite and Mishra (2016); Jia et al. (2016)). Deployment of multiple sensors provides more advantages over a single node. In particular, each sensor mostly receives incomplete observations (measurements) because of the noisiness of an environment and inaccuracy inherent to the sensor devices. Thanks to the use of multiple sensors one might obtain more accurate estimation of the measured value through the information fusion. In other words, multi-sensor networks can be used to reduce uncertainties.

Kalman filters (KFs) without doubt are one of the most common algorithms which the tracking systems rely on Kalman et al. (1960); Uhlmann (1992). For multi-target tracking it has become as much important as it has been for single target tracking. Nevertheless, these algorithms address not all aspects of the overall problem. When a tracking system is required to track a large number of targets, the computation and communication loads arise. One possible solution is to use a distributed scheme. In order to do that, consensus algorithms have been adopted. In Olfati-Saber (2007); Cattivelli and Sayed (2010); Di Paola et al. (2015) the authors suggested distributed estimation schemes based on Kalman filtering. In the follow-up works the researchers address the issues inherent to sensor networks such as limited sensing capability, heterogeneity, asynchronous messaging, energy efficiency Olfati-Saber and Sandell (2008); Petitti et al. (2011); Giannini et al. (2013); Yang and Shi (2012). In Yu (2017) distributed target tracking algorithm in the presence of data association uncertainty is suggested. The authors utilized the maximum a posteriori approach to deal with the data association process.

Besides of the consensus approach, researches have suggested to use the stochastic approximation algorithms. In Spall (2012) considered a modification of stochastic approximation procedure based on a cyclic approach. The essence of the approach is that the parameter vector is divided into several subvectors, which then is sequentially updated while holding the remaining parameters at their most recent values. In Botts et al. (2016) the authors consider a stochastic multi-agent and multi-target surveillance problem and apply to it a cyclic stochastic optimization algorithm.

The research described in this paper is built upon our previous works regarding stochastic approximation algorithms (see Amelin et al. (2013); Granichin and Amelina (2015); Granichin and Erofeeva (2018)). We have examined this kind of algorithms in different optimization problems (i.e., (non)-stationary, (non)-constrained) along with the presence of noise in observations. In this paper we examine a cyclic stochastic approximation method with a perturbation on the input in the resource constrained problem. In contrast with the previous works, in this paper we consider an estimation process in possibly large networks. In this case we need to take into account the available computation and communication resources of the sensor network. In particular, we propose a distributed algorithm for state estimating (i.e., positions) of multiple moving targets by the sensor network. After that, we get an upper bound of residual between estimates for the suggested algorithm. Note that we don't consider the data association problem occurring in multi-

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ple target tracking applications. We assume it has already been done by some other method.

The remainder of the paper is organized as follows. Section II presents the problem statement. In Section III, we describe the cyclic approach. The main result is given in Section IV, where we present a distributed algorithm for multiple target tracking and analyze its properties. Results of the experiments are shown in Section V. Sections VI concludes the paper.

The notation used in this paper can be described as follows. Upper and lower bold face letters are used for matrices and column vectors, respectively. $\mathrm{E}\{\cdot\}$ is the expectation operation. $(\cdot)^{\mathrm{T}}$ denotes transposition. $|\mathscr{U}|$ denotes the cardinality of the set $\mathscr{U}$. $\|\cdot\|$ is the Euclidean norm. $\mathrm{tr}\{\cdot\}$ is the matrix trace operator.

2. PROBLEM STATEMENT

Consider a distributed network consisting of n sensors, which observe m moving objects. Let $N = \{1,2,\ldots,n\}$ be a set of sensors, $M = \{1,2,\ldots,m\}$ be a set of objects, $\mathbf{s}_t^j \in \mathbb{R}^m$ be a state of the sensor j, $j \in N$ at time instant t, $\mathbf{r}_t^i \in \mathbb{R}^p$ be a state of the object i, $i \in M$ at time instant t.

The sensors estimate the state $\mathbf{r}_t^i$ of the object i based on the measurements received in accordance with the following observation model

$$\mathbf{z}_t^{i,j} = \varphi(\mathbf{s}_t^j, \mathbf{r}_t^i) + \varepsilon_t^{i,j}, \tag{1}$$

where $\mathbf{z}_t^{i,j} \in \mathbb{R}^q$ is a measurement of the state of the object i available to the sensor j at time instant t, $\varphi(\cdot,\cdot): \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}^q$ is an observation function, which depends on the current state of the object i and sensor j, $\{\varepsilon_t^{i,j}\}$ is the additive external noise with zero mean $\mathrm{E}\varepsilon_t^{i,j} = 0$ and the error covariance matrix $\mathrm{E}\varepsilon_t^{i,j}(\varepsilon_t^{i,j})^{\mathrm{T}} = \Sigma_t^{i,j}$.

We assume that for any $i \in M$, $j \in N$ and independent centered $\varepsilon_t^{i,j}$ with the error covariance matrix $\Sigma_t^{i,j}$ there exists the inverse function $\varphi^{-1}(\mathbf{s}_t^j, \cdot): \mathbb{R}^q \to \mathbb{R}^p$:

$$\varphi^{-1}(\mathbf{s}_t^j, \varphi(\mathbf{s}_t^j, \mathbf{r}_t^i) + \varepsilon_t^{i,j}) = \mathbf{r}_t^i + \xi_t^{i,j}, \tag{2}$$

where $\xi_t^{i,j}$ is an independent component with zero mean $E\xi_t^{i,j} = 0$, the error covariance matrix $\mathrm{E}\xi_t^{i,j}(\xi_t^{i,j})^{\mathrm{T}} = \Xi_t^{i,j}$ and the bounded fourth central moment $\mathrm{E}\|\xi_t^{i,j}\|^4 \le M_4$.

Note that the measurements received by a single sensor might not be enough to reconstruct the state of an object. In this case a sequence of measurements collected by the sensor itself or through other sensors is usually utilized. Nevertheless, due to availability of technologically advanced equipment it is possible to satisfy the assumption (2). If there is no such single-valued inverse function, but there exists a subspace corresponding to $\mathbf{z}_t^{i,j} - \mathbf{U}_t^{i,j}\mathbf{r}_t^i = 0$, where $\mathbf{U}_t^{i,j}$ is a matrix mapping the state into the measurement, then we are able to estimate the true state on this subspace.

In addition, we assume that with some probability p_σ the mean value of $\mathrm{tr}(\Xi_t^{i,j})$ is less than some threshold value $(\bar{\sigma}_{\min})^2 > 0$ and its mean value is equal to $(\bar{\sigma}_t^{i,j})^2$ if the threshold $(\bar{\sigma}_{\min})^2$ is exceeded.

We denote by $\theta_t = col(\mathbf{r}_t^1, \ldots, \mathbf{r}_t^m)$ the joint vector of all object states. Let $\widehat{\mathbf{r}}_t^i$ be an estimate of the state of object i at time instant t and $\widehat{\theta}_t = col(\widehat{\mathbf{r}}_t^1, \ldots, \widehat{\mathbf{r}}_t^m)$ be the joint vector of all estimates.

The problem of estimating unknown states of objects can be formulated as the problem of minimizing the functional

$$\bar{F}_t(\widehat{\theta}_t) = \frac{1}{2}\sum_{i\in M} \|\mathbf{r}_t^i - \widehat{\mathbf{r}}_t^i\|^2 \to \min_{\widehat{\theta}_t}. \tag{3}$$

In Granichin and Erofeeva (2018) we have shown that (3) might be considered as the mean risk functional. To solve the problem (3) we are going to use the algorithm and results from Granichin and Amelina (2015). However, we need to adjust it to the distributed case.

2.1 Mean-Risk Optimization

Let $(\Omega, \mathscr{F}, \mathbb{P})$ be the underlying probability space, where Ω is a set of all possible results (outcomes) of an experiment, $\mathscr{F}$ is a σ-algebra of subsets of Ω, and $\mathbb{P}$ is a probability measure function, E is a mathematical expectation, $\mathbb{W}$ is some set (i.e., $\mathbb{W} = \mathbb{N}$ or $\mathbb{W} \subset \mathbb{R}^p$).

Consider a set of differentiable functions $\{\bar{f}_w(\theta)\}_{w\in\mathbb{W}}$, $\bar{f}_w(\theta): \mathbb{R}^d \to \mathbb{R}$. Let $\mathbf{x}_1, \mathbf{x}_2, \ldots$ be the set of observation points chosen by the experimenter. For each $t = 1,2,\ldots$ we get the measurements (observations) $y_1, y_2, \ldots$ of $\bar{f}_w(\cdot)$ with the additive external noise v_t

$$y_t = \bar{f}_{w_t}(\mathbf{x}_t) + v_t, \tag{4}$$

where v_t represents the error emerging due to random quantities such as imperfect state estimates, $\{w_t\}$ is an uncontrollable sequence, $w_t \in \mathbb{W}$. In some cases v_t can be considered as a part of the vector w_t. However, in the general case it is better to separate them since the noise in the measurements is a sensor property, and the random vector w_t is a property of the optimized system.

We denote by $\mathscr{F}_{t-1}$ the σ-algebra of probabilistic events generated by those quantities from $w_0, \ldots, w_{t-1}, x_0, \ldots, x_{t-1}, v_0, \ldots, v_{t-1}$, which are random, $\mathrm{E}_{\mathscr{F}_{t-1}}$ is a symbol for conditional mathematical expectation with respect to σ-algebra $\mathscr{F}_{t-1}$. We assume that if w_t is random, then $\bar{f}_{w_t}(\theta)$ as a function of w_t is measurable for each θ with respect to σ-algebra $\mathscr{F}_{t-1}$.

The problem is to find optimal θ_t that minimizes mean risk functional

$$\bar{F}_t(\theta) = E_{\mathscr{F}_{t-1}}\bar{f}_{w_t}(\theta) \to \min_{\theta}. \tag{5}$$

subject to linear constraints

$$\mathbf{H}\theta = \mathbf{q}_{t-1} \tag{6}$$

with matrix $\mathbf{H}$ of dimension $l \times d$ and vectors $\mathbf{q}_{t-1} \in \mathbb{R}^l$, $0 \le l < d$ (if $l = 0$ we assume that there are no constrains).

If $\mathrm{rank}\mathbf{H} = l$ then there exists linear function $h: \mathbb{R}^d \to \mathbb{R}^{d-l}$ and its reverse function $g_t: \mathbb{R}^{d-l} \to \mathbb{R}^d$ such as

$$\mathbf{x} = g_t(h(\mathbf{x})),\ \forall \mathbf{x} \in \mathbb{Q}_t = \{\mathbf{x} : \mathbf{H}\mathbf{x} = \mathbf{q}_{t-1}\}.$$

If we choose $\mathbb{W} = \otimes_{i=1}^m \otimes_{j=1}^n \mathbb{R}^q \otimes_{j=1}^n \mathbb{R}^p$ and denote $w_t = col(\ldots, \varepsilon_t^{i,j}, \ldots, \mathbf{s}_t^j, \ldots)$, $\mathbf{x}_t = \widehat{\theta}_t$,

$$\bar{f}_{w_t}(\mathbf{x}_t) = \frac{K}{2n}\sum_{j\in N}\sum_{i\in M} \|\varphi^{-1}(\mathbf{s}_t^j, \mathbf{z}_t^{i,j}) - \widehat{\mathbf{r}}_t^i\|^2/(\sigma_t^{i,j})^2,$$

where $K = \mathrm{p}_\sigma(\bar{\sigma}_{\min})^2 + (1-\mathrm{p}_\sigma)\sum_{j\in N}(\bar{\sigma}_t^{i,j})^2$, the corresponding summands in the sum are assumed to be zero if $(\sigma_t^{i,j})^2 = \infty$ and $(\sigma_t^{i,j})^2 = \max\{\mathrm{tr}(\Xi_t^{i,j})\}$ otherwise, then (3) is a mean risk functional similar to (5), since $\xi_t^{i,j}$ is an independent and centered component.

2.2 Distributed Case

In distributed optimization it is assumed that for any $w \in \mathbb{W}$ the function $\bar{f}_w(\theta)$ is separable with respect to the function itself or splitting of θ into n subvectors, meaning that:

$$\bar{f}_w(\theta) = \sum_{j=1}^{n} f_w^j(\theta^j), \tag{7}$$

where $\theta^j \in \mathbb{R}^d$ is a copy of θ for each $j = 1, \ldots, n$ or $\theta^j \in \mathbb{R}^{d^j}$ is a subvector of $\theta = col(\theta^1, \ldots, \theta^n)$, respectively. In the first case, each sensor i will generate its own estimate of an optimal solution to the problem based on the available local information. Then the optimal solution set is $\Theta^\star = \{\theta^j \in \mathbb{R}^d \mid \sum_{j=1}^n f_w^j(\theta^j) = \bar{f}_w^\star\}$. In the second case, each sensor j is associated with some part $\theta^{j^\star}$ of a global optimal solution $\theta^\star$.

Now, we rewrite the problem (5) in a distributed way as follows:

$$\bar{F}_t(\theta) = E_{\mathscr{F}_{t-1}} \sum_{j=1}^{n} f_{w_t}^j(\theta^j) = \sum_{j=1}^{n} F_t^j(\theta^j) \to \min_{\theta}, \tag{8}$$

subject to linear constraints

$$\mathbf{H}^j \theta^j = \mathbf{q}_{t-1}^j \tag{9}$$

with matrix $\mathbf{H}^j$ of dimension $l \times d$ and vectors $\mathbf{q}_{t-1}^j \in \mathbb{R}^l$, $0 \le l < d$.

Next, we specify the observation function $\varphi(\cdot,\cdot)$.

2.3 Observation Model

We consider a 2D-plane, in which the state of the object i is $\mathbf{r}_t^i = [r_t^{i,1}\ r_t^{i,2}]^T$ and the state of the sensor j is $\mathbf{s}_t^j = [s_t^{j,1}\ s_t^{j,2}]^T$. Suppose the sensors are able to determine the angle and distance to the objects, then:

$$\varphi(\mathbf{s}_t^j, \mathbf{r}_t^i) = \begin{bmatrix} \psi(\mathbf{s}_t^j, \mathbf{r}_t^i) \\ \rho(\mathbf{s}_t^j, \mathbf{r}_t^i) \end{bmatrix} \in \mathbb{R}^2, \tag{10}$$

where

$$\psi(\mathbf{s}_t^j, \mathbf{r}_t^i) = \operatorname{arctg}\left[\frac{r_t^{i,1} - s_t^{j,1}}{r_t^{i,2} - s_t^{j,2}}\right] \tag{11}$$

is the angle to the object i,

$$\rho(\mathbf{s}_t^j, \mathbf{r}_t^i) = \sqrt{\left(r_t^{i,1} - s_t^{j,1}\right)^2 + \left(r_t^{i,2} - s_t^{j,2}\right)^2} \tag{12}$$

is the distance to the object i.

In this case, the inverse function $\varphi^{-1}(\mathbf{s}_t^j, \cdot)$ is as follows

$$\varphi^{-1}(\mathbf{s}_t^j, \mathbf{z}_t^{i,j}) = \mathbf{s}_t^j + \begin{bmatrix} z_t^{i,j,2} \sin z_t^{i,j,1} \\ z_t^{i,j,2} \cos z_t^{i,j,1} \end{bmatrix}, \tag{13}$$

where $z_t^{i,j,1}$ and $z_t^{i,j,2}$ are the first and second coordinates of the vector $\mathbf{z}_t^{i,j}$, respectively. If the error covariance matrices $\varepsilon_t^{i,j}$ are equal to $\Sigma_t^{i,j} = \begin{bmatrix} \sigma_\psi^2 & 0 \\ 0 & (z_t^{i,j,2}\sigma_\rho)^2 \end{bmatrix}$, then the error covariance matrix of $\xi_t^{i,j}$ is

$$\Xi_t^{i,j} = R(z_t^{i,j,1}) \begin{bmatrix} (z_t^{i,j,2}\sigma_\psi)^2 & 0 \\ 0 & (z_t^{i,j,2}\sigma_\rho)^2 \end{bmatrix} R(z_t^{i,j,1})^{\mathrm{T}}, \tag{14}$$

where $R(\psi) = \begin{bmatrix} \sin\psi & -\cos\psi \\ \cos\psi & \sin\psi \end{bmatrix}$ is the rotation matrix through the angle ψ. Note that $\operatorname{tr}(\Xi_t^{i,j}) = (z_t^{i,j,2}\sigma_\psi)^2 + (z_t^{i,j,2}\sigma_\rho)^2$.

3. CYCLIC APPROACH

In Spall (2012); Cuevas (2017) the authors suggested a cyclic stochastic approximation algorithm to find the estimate of the vector θ_t and provided theoretical results for the case of decreasing to zero step sizes. However, the use of decreasing to zero step sizes in the problem (5), where it is required to track changes of the estimating parameter θ_t, may significantly affect the convergence. Such problem actualizes the study of the properties of cyclic stochastic approximation algorithms with nondecreasing step size, which is the focus of this paper.

The cyclic approach allows us to move from a centralized formulation of the problem to a distributed one. The peculiarity of the cyclic approach application is that an unknown vector θ_t is divided into several subvectors, herewith at time instant t only a selected subvector is updated. Nevertheless, it is implied that for some time interval each subvector will be fully updated.

Let us consider more closely how the cyclic approach might be applied to the early described problem. We divide the time axis by a sequence of cycles of length $2k$: $2(T-1)k+1, 2(T-1)k+2, \ldots, 2Tk$ and on each of the cycles we partition the set of indices $\mathbb{D} = \{1, \ldots, d\}$ into k disjoint subsets $\mathbb{I}_u$, $u = 1, \ldots, k$. The subsets $\mathbb{I}_u$ derive the "active" parameters at time instants $t = 2(T-1)k + 2u - 1$ $t = 2(T-1)k + 2u$, $u = 1, \ldots, k$ from the whole parameters set. This subsets should satisfy the following conditions

$$\bigcup_{u=1}^{k} \mathbb{I}_u = \mathbb{D}, \quad \mathbb{I}_{u'} \bigcap \mathbb{I}_{u''} = \emptyset \quad \text{for } u' \neq u''. \tag{15}$$

For each $t = 1, 2, \ldots$ we define diagonal matrices A_t, forming a sparse vector $A_t \mathbf{x}_t$ from $\mathbf{x}_t$ with zeros at those places whose indices do not belong to $\mathbb{I}_{(t \bmod (2k)) \div 2}$. Here mod is an operation of taking the remainder of the division of one number by another, $\div$ is an integer division. Based on the cyclic sequence of matrices $\{A_t\}$ we define a polynomial

$$\mathscr{A}(\lambda) = \sum_{u=1}^{k} A_{2kT+2u} \lambda^u,$$

which later will be used along with the operation of shifting the index $\lambda \theta_t = \theta_{t-2k+2}$.

Taking into account the notation introduced, the obtained observations $y_1, y_2, \ldots$ can be represented as follows

$$y_t = f_{w_t}(A_t \mathbf{x}_t) + v_t. \tag{16}$$

Let us define the adjacency matrix $B_t = [b_t^{i,j}]$, where $b_t^{i,j} > 0$ if the sensor j is able to track the object i and $b_t^{i,j} = 0$ otherwise. Similarly, we introduce the interaction matrix $C_t = [c_t^{j,k}]$, where $c_t^{j,k} > 0$ if the sensor j is able to communicate with the sensor $k \in N$ and $c_t^{j,k} = 0$ otherwise. We denote by $N_t^j = \{j : c_t^{j,k} > 0\} \subset N$ a set of "neighbors" of the sensor j. Let $M_t^j \subset M$ be a set of objects of the sensor j, which he can observe itself at time instant t or receive measurements from its neighbors.

We consider two types of restrictions placed on the sensor network functioning. The first restriction is that each sensor is able to exchange data only with a certain number of "neighbors", i.e. we assume that the following constraints arise

$$|N_t^j| \le n_{\max}^j. \tag{17}$$

In a real environment, this restriction may arise, for example, if the number of dedicated communication channels is limited or if sensors are unable to send data to a distance greater than a certain maximum. The second restriction is related to the maximum allowable number of objects to be tracked by sensor j itself or through information from its "neighbors" at time instant t

$$|M_t^j| \leq m_{\max}^j, \tag{18}$$

In turn, this restriction may be associated with the limited throughput of the communication channel. Note that we form the subsets of M_t^j by varying the coefficients of the adjacency matrix B_t.

Let matrices B_t and C_t satisfy the conditions (17) and (18). Moreover, each sensor j, $j \in N$ observe the objects belonging to some set $\mathbb{D}^j$ under the conditions:

$$\bigcup_{u=1}^{k^j} \mathbb{I}_u^j = \mathbb{D}^j, \quad \mathbb{I}_{u'}^j \bigcap \mathbb{I}_{u''}^j = \emptyset \quad \text{for } u' \neq u''. \tag{19}$$

We denote by A_t^j matrices that form the sparse vectors $\widehat{\theta}_t$.

With the notation introduced, the functional (3) can be rewritten as (8)

$$f_t^j(\widehat{\theta}_t^j) = \frac{K}{2n} \sum_{i \in M_t^j} \|\varphi^{-1}(\mathbf{s}_t^j, \mathbf{z}_t^{i,j}) - \widehat{\mathbf{r}}_t^i\|^2 / (\sigma_t^{i,j})^2, \tag{20}$$

since we can assume that $(\sigma_t^{i,j})^2 = \infty$ if the sensor j does not receive any information about the object i at time instant t.

4. MAIN RESULT

We are going to use General Cyclic Simultaneous Perturbation Stochastic Approximation (GCSPSA) with a perturbation on the input and linear constraints to track changes of θ_t, see Granichin and Erofeeva (2018); Erofeeva (2018).

4.1 Distributed Cyclic Estimation Algorithm

Let $\widehat{\theta}_0 \in \mathbb{R}^d$ be a nonrandom initial vector, Δ_T, $T = 0, 1, \ldots,$ be an observed sequence of independent random vectors in $\mathbb{R}^d$, called *the simultaneous perturbation vectors*, which are equal to ± 1 with probability $\frac{1}{2}$. The following Algorithm 1 should be carried out on each sensor j, $j \in N$.

Algorithm 1. Distributed State Estimation Based on GCSPSA

Input: $\alpha^j > 0$, $\beta^j > 0$, $n_{\max}^j$, $m_{\max}^j$

Output: $\widehat{\theta}_t^j$

Initialization: Set the counter index $T^j = 0$. Select an initial guess $\widehat{\theta}_0^j \in \mathbb{R}^d$. Form a sequence of matrices $\{A_t^j\}$ such that the conditions (17)-(19) are satisfied.

1. Set $T^j \leftarrow T^j + 1$.

2. Generate the random vector $\Delta_{T^j}^j$ according to the Bernoulli distribution of i.i.d. components that are equal to ± 1 with probability $\frac{1}{2}$.

3. For $u \leftarrow 1$ to k^j repeat:

3.1 Set $t \leftarrow 2T^j + 2u$.

3.2 Form an observation point $\mathbf{x}_{t-1}^j$. If $a_{t-1}^{j,l} > 0$ then $\mathbf{x}_{t-1}^{j,l} \leftarrow \widehat{\theta}_{2T^j}^{j,l} - \beta^j \Delta_{T^j}^{j,l}$ and $\mathbf{x}_{t-1}^{j,l} \leftarrow 0$ otherwise.

3.3 Get measurements $\mathbf{z}_{t-1}^{i,j}$, $i \in M_{t-1}^j$.

3.4 Calculate $y_{t-1}^{j,-}$ using (20): $y_{t-1}^{j,-} \leftarrow f_t^j(\mathbf{x}_{t-1}^j)$.

3.5 Form an observation point $\mathbf{x}_t^j$. If $a_t^{j,l} > 0$ then $\mathbf{x}_t^{j,l} \leftarrow \widehat{\theta}_{2T^j}^{j,l} + \beta^j \Delta_{T^j}^{j,l}$ and $\mathbf{x}_t^{j,l} \leftarrow 0$ otherwise.

3.6 Get measurements $\mathbf{z}_t^{i,j}$, $i \in M_t^j$.

3.7 Calculate $y_t^{j,+}$ using (20): $\widehat{\nabla}_t^j \leftarrow A_t^j \Delta_{T^j}^j \frac{y_t^{j,+} - y_{t-1}^{j,-}}{2\beta^j}$.

3.8 Calculate pseudogradient: $\widehat{\nabla}_t^j \leftarrow A_t^j \Delta_{T^j}^j \frac{y_t^{j,+} - y_{t-1}^{j,-}}{2\beta^j}$.

3.9 Get the new estimation $\widehat{\theta}_t^j \leftarrow \widehat{\theta}_{t-1}^j - \alpha^j \widehat{\nabla}_t^j$.

Go to step 1.

4.2 Supporting Theorem and Assumptions

This subsection presents Theorem 1 and assumptions for the general cyclic SPSA procedure with a perturbation on the input, which are provided in Erofeeva (2018). The proof of Theorem 1 is based on the results in Granichin and Erofeeva (2018), where we have considered a special case, i.e. when there are no constraints. In the next subsection we are going to formulate Theorem 2 based on Theorem 1 for a particular problem described in Section II.

Let us provide Assumptions about disturbances and functions $\bar{f}_w(\mathbf{x})$, $\bar{F}_t(\mathbf{x})$:

As1. For the minimum points θ_t of functions $\bar{F}_t(\cdot)$ and the gradients of the functions $\tilde{f}(A_t\mathbf{x}) = \bar{f}_{w_t}(g_t(A_t\mathbf{x}))$ the inequalities hold
$\forall \mathbf{x} \in \mathbb{R}^d \;\; (\mathbf{x} - h(\theta_t))^{\mathrm{T}} A_t^{\mathrm{T}} \mathrm{E}_{\mathscr{F}_{t-1}} \nabla \tilde{f}_{w_t}(A_t\mathbf{x}) \geq \mu \|A_t(\mathbf{x} - h(\theta_t))\|^2$
with a constant $\mu > 0$.

As2. $\forall w \in \mathbb{W}$ the gradient $\nabla \tilde{f}_{w_t}(A_t\mathbf{x})$ satisfies the Lipschitz condition: $\forall \mathbf{x}', \mathbf{x}'' \in \mathbb{R}^d$

$$\|\nabla \tilde{f}_{w_t}(A_t\mathbf{x}') - \tilde{f}_{w_t}(A_t\mathbf{x}'')\| \leq M \|A_t(\mathbf{x}' - \mathbf{x}'')\|$$

with a constant $M \geq \mu$.

As3. The gradient $\nabla \tilde{f}_{w_t}(A_t\mathbf{x})$ is uniformly bounded in the mean-squared sense at the minimum points θ_t: $\|\mathrm{E}\nabla \bar{f}_{w_t}(A_t h(\theta_t))\| \leq c_1$, $\mathrm{E}\|\nabla \bar{f}_{w_t}(A_t h(\theta_t))\|^2 \leq c_2$, $\mathrm{E}(\nabla \bar{f}_{w_t}(A_t h(\theta_t)))^{\mathrm{T}} \nabla \bar{f}_{w_{t-1}}(A_t \cdot \cdot h(\theta_{t-1})) \leq c_2$ ($c_1 = c_2 = 0$, if w_t is not a random parameter, i.e. $\bar{f}_{w_t}(\mathbf{x}) = \bar{F}_t(\mathbf{x})$).

As4. The drift is bounded: for $\eta_t = A_t(h(\theta_t - \theta_{t-1}))$ the condition $\|\eta_t\| \leq \delta_\theta < \infty$ is met, or $\mathrm{E}\|\eta_t\|^2 \leq \delta_\theta^2$ and $\mathrm{E}\|\eta_t\|\|\eta_{t-1}\| \leq \delta_\theta^2$ are met if a sequence $\{w_t\}$ is random.

As5. The rate of drift is bounded in such a way that for any arbitrary point $\forall \mathbf{x} \in \mathbb{R}^d$: $\mathrm{E}_{\mathscr{F}_{t-2}}(\tilde{f}_{w_t}(A_t\theta_t) - \tilde{f}_{w_{t-1}}(A_t\theta_{t-1}))^2 \leq c_3\|A_t(\mathbf{x} - h(\theta_{t-2}))\| + c_4$.

As6. Sequential differences of the observation noise are limited: $|v_{2t} - v_{2t-1}| \leq c_v < \infty$ or $\mathrm{E}(v_{2t} - v_{2t-1})^2 \leq c_v^2$, if the sequence $\{v_t\}$ is random.

As7. For any $T = 0, 1, \ldots,$ if v_t is random, then the vector Δ_T and the differences $v_{2kT+2} - v_{2kT+1}, \ldots, v_{2k(T+1)} - v_{2k(T+1)-1}$ are independent; if w_t is random, then the vector Δ_T and $w_{2kT+1}, \ldots, w_{2k(T+1)}$ are independent.

Theorem 1. **If** Assumptions 1–7 hold, the constant α^j is sufficiently small and the conditions (15) are fulfilled: $\alpha \in (0; \mu/\gamma)$

if $(\mu)^2 < 2\gamma$, or $\alpha \in \left(0; \frac{\mu-\sqrt{(\mu)^2-2\gamma}}{2\gamma}\right) \cup \left(\frac{\mu+\sqrt{(\mu)^2-2\gamma}}{2\gamma}; \mu/\gamma\right)$ otherwise, **then** the sequence of estimates $\{\widehat{\theta}_{2kT}\}_{T=0}^{\infty}$ provided by the GCSPSA algorithm has an asymptotically efficient upper bound which equals to: $\forall \varepsilon > 0\ \exists\ \bar{t} : \forall t > \bar{t}$

$$\sqrt{\mathrm{E}\|h(\widehat{\theta}_t - \mathscr{A}(\lambda)\theta_t)\|^2} \le \frac{\sqrt{k}\left(b + \sqrt{b^2 + ml}\right)}{m} + \varepsilon, \qquad (21)$$

where $\gamma = 3d(M^2 d + \frac{c_3}{\beta})$, $m = 2(\mu - \alpha\gamma)$, $b = 2\beta M d\sqrt{d}(1 + 6\alpha M d) + \delta_\theta(M + 2\mu + 6\alpha M^2 d^2)$, $\bar{l} = 2\alpha d(c_v^2 + 3(\frac{c_4}{\beta} + d(c_2 + M^2(\delta_\theta + 2\beta\sqrt{d})^2))) + 2\delta_\theta(4\beta M d\sqrt{d} + M\delta_\theta + c_1 + 3\mu\delta_\theta^2)$, $l = \bar{l} + 2bk\sqrt{k}\delta_\theta + \frac{1-\alpha m}{\alpha}\delta_\theta^2$.

Proof. The proof is given in Erofeeva (2018).

4.3 Upper Bound of Residuals of Estimation

This subsection presents a theorem that provide estimation properties for Algorithm 1.

Theorem 2. **If** Assumptions 1–7 hold, the constant α^j is sufficiently small, the drift $\|\mathbf{r}_t^i - \mathbf{r}_{t-1}^i\| \le \sigma_{\mathbf{r}}^i$, $i \in M$ is bounded and the following conditions are fulfilled: (2) is for the observation model, (17)–(19) are for the matrix sequences $\{B_t\}$, $\{C_t\}$ and $\{A_t^j\}$, $j \in N$: $\alpha^j \in (0; \mu^j/\gamma^j)$ if $(\mu^j)^2 < 2\gamma^j$, or $\alpha^j \in \left(0; \frac{\mu^j-\sqrt{(\mu^j)^2-2\gamma^j}}{2\gamma^j}\right) \cup \left(\frac{\mu^j+\sqrt{(\mu^j)^2-2\gamma^j}}{2\gamma^j}; \mu^j/\gamma^j\right)$ otherwise,

then the sequence of estimates $\{\widehat{\theta}^j_{2k^jT^j}\}_{T^j=0}^{\infty}$ provided by the Algorithm 1 has an asymptotically efficient upper bound which equals to: $\forall \varepsilon^j > 0\ \exists\ \bar{t}^j : \forall t > \bar{t}^j$

$$\sqrt{\mathrm{E}\|h(\widehat{\theta}_t^j - \mathscr{A}(\lambda)\theta_t)\|^2} \le \frac{\sqrt{k^j}\left(b^j + \sqrt{(b^j)^2 + m^j l^j}\right)}{m^j} + \varepsilon^j,$$

where $\mu^j = \frac{K}{2n\max_{i,t}(\sigma_t^{i,j})^2}$, $M^j = \frac{K}{2n\min_{i,t}(\sigma_t^{i,j})^2}$, $\gamma^j = 3d^2(M^j)^2$, $m^j = 2(\mu^j - \alpha\gamma^j)$, $\delta_\theta^j = k^j \max_{i,t}\sum_{i\in M_t^j}\delta_{\mathbf{r}}^i$, $b^j = 2\beta M^j d\sqrt{d}(1 + 6\alpha M^j d) + \delta_\theta^j(M^j + 2\mu^j + 6\alpha(M^j)^2 d^2)$, $\bar{l}^j = 6d\frac{\alpha}{\beta}\max_t \frac{K}{2n}\cdot$

$$\cdot\sum_{i\in M_t^j}\left(\frac{M_4}{(\sigma_t^{i,j})^4} + \frac{M_4}{(\sigma_{t-1}^{i,j})^4} - 2\right) + 6d^2\left(\frac{K}{2n}\max_{i,t}\frac{\mathrm{tr}(\Xi_t^{i,j})}{(\sigma_t^{i,j})^2} +\right.$$

$+(M^j)^2(\delta_\theta^j + 2\beta\sqrt{d})^2) + 2\delta_\theta^j(4\beta M^j d\sqrt{d} + M^j\delta_\theta^j + 3\mu^j(\delta_\theta^j)^2)$, $l^j = \bar{l}^j + 2b^j k^j\sqrt{k^j}\delta_\theta^j + \frac{1-\alpha m^j}{\alpha}(\delta_\theta^j)^2$.

Proof. The proof is given in Appendix.

Remarks: Due to time-varying nature of the estimated parameter in the target tracking problem, computations need to be performed faster than the rate at which the parameter evolves.

Assumptions 1 and 2 imply that the function have to be convex and the gradient have to be Lipschitz, respectively. Assumption 3 is general. Assumptions 4 and 5 are related to the sampling time and the speed of an object. Assumptions 6 and 7 state that the noise have to be bounded and independent from the simultaneous perturbation vector generated by the algorithm.

5. EXPERIMENTS

Suppose six objects are moving in a square area of 300×300 km^2. The objects have the same initial velocity, which is equal to 2500 km/h. The velocities of each object slightly change over time.

Six stationary sensors are randomly located in the area of interest. Each sensor receives noisy measurements. The level of errors in the measurements is set to 5% for distances and 0.5 degrees for angles.

Objects and sensors have a common coordinate system with axes x^1 and x^2. Objects start their movement at the points with coordinates $\mathbf{r}_0^1 = [270, 295]^{\mathrm{T}}$; $\mathbf{r}_0^2 = [240, 290]^{\mathrm{T}}$; $\mathbf{r}_0^3 = [210, 285]^{\mathrm{T}}$; $\mathbf{r}_0^4 = [180, 280]^{\mathrm{T}}$; $\mathbf{r}_0^5 = [150, 275]$; $\mathbf{r}_0^6 = [120, 270]^{\mathrm{T}}$. For each sensor $j \in N$ we set $\alpha^j = 0{,}05$ $\beta^j = 0{,}03$. The initial guess $\widehat{\theta}_0^j$ is randomly assigned from the interval $[299, 300]^{\mathrm{T}}$. Let the maximum number of targets that each sensor is able to observe be $m_{\max}^j \in \{1, \ldots, 6\}$ and the maximum possible number of "neighbors" be $n_{\max}^j = 2$.

Figures 1-2 present the simulation results for a chosen value of $m_{\max}^j$. Here, err_t is an estimation error at time t. With a low number of possible tracking objects, the rate of convergence of the estimate to the real value is significantly slowed down. In turn, with the maximum possible value of $m_{\max}^j$, the algorithm demonstrates the best convergence.

Based on the simulation results, we can say that it is possible not to use all available sensors to track each target. As a result it is possible to use the resources of the sensor network more rationally and increase its tracking characteristics.

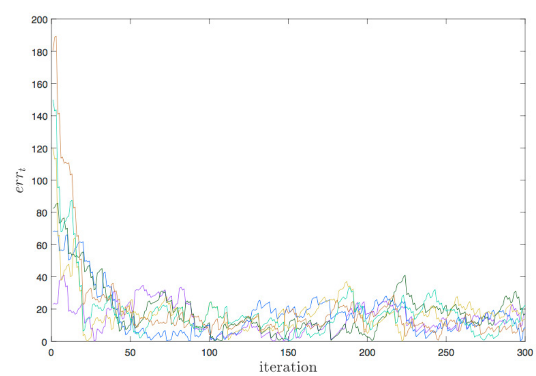

Fig. 1. An estimation error for $m_{\max}^j = 3$

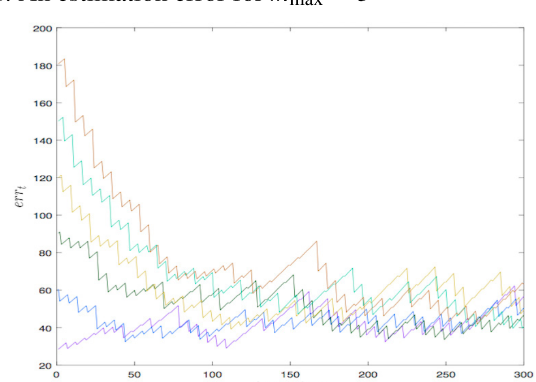

Fig. 2. An estimation error for $m_{\max}^j = 1$

6. CONCLUSIONS

Algorithm 1 provides an efficient solution for multi-target tracking in the sense of memory and computation. If the conditions of Theorem 2 are hold, we will be able to estimate the states of the moving objects despite the presence of the sensor network limitations. However, it comes for a price. If we are able to track only a small amount of all targets that have to be tracked, the rate of convergence becomes slow. In this case, we may find a trade-off between the rate of convergence and the possible amount of targets, which the system tracks at some time interval. In follow-up works we are going to study how changes to the network topology affect the performance of the algorithm.

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Appendix A. THE PROOF OF THEOREM 2

To prove Theorem 2, it suffices to verify if Assumptions 1–7 of Theorem 1 for functions $F_t^j(A_t^j\mathbf{x})$ and $f_t^j(A_t^j\mathbf{x})$ are fulfilled. In order to do this, we first compute the components of the gradient of the function $F_t^j(A_t^j\mathbf{x})$:

$$\frac{\partial}{\partial x^{i,l}}\nabla F_t^j(A_t^j\mathbf{x}) = E_{\mathscr{F}_{t-1}}\frac{\partial}{\partial x^{i,l}}\nabla f_t^j(A_t^j\mathbf{x}) =$$
$$= E_{\mathscr{F}_{t-1}}\sum_{i\in M_t^j}\frac{K(x^{i,l}+\xi^{i,l}-r^{i,l})}{2n(\sigma_t^{i,j})^2}\cdot 1 = \sum_{i\in M_t^j}\frac{Kx^{i,l}}{2n(\sigma_t^{i,j})^2} - r^{i,l}.$$

From the last formula it follows that

Assumption 1 holds if $\mu^j = \frac{K}{2n\max_{i,t}(\sigma_t^{i,j})^2}$,

Assumption 2 holds if $M^j = \frac{K}{2n\min_{i,t}(\sigma_t^{i,j})^2}$,

Assumption 3 holds if $c_1^j = 0$ $c_2^j = \frac{K}{2n}\max_{i,t}\frac{\mathrm{tr}(\Xi_t^{i,j})}{(\sigma_t^{i,j})^2}$,

Assumption 4 holds if $\delta_\theta^j = k^j \max_{i,t}\sum_{i\in M_t^j}\delta_{\mathbf{r}}^i$.

Let us now verify the Assumptions 5-6. Assumptions 5 holds if $c_3^j = 0$ and $c_4^j = \max_t \frac{K}{2n}\sum_{i\in M_t^j}\left(\frac{M_4}{(\sigma_t^{i,j})^4}+\frac{M_4}{(\sigma_{t-1}^{i,j})^4}-2\right)$. For the corresponding difference we have

$$\mathrm{E}_{\mathscr{F}_{t-2}}(\bar{f}_{w_t}^j(A_t\theta_t) - \bar{f}_{w_{t-1}}^j(A_t\theta_{t-1}))^2 =$$
$$= \frac{K}{2n}\mathrm{E}_{\mathscr{F}_{t-2}}\sum_{i\in M_t^j}\left(\|\frac{\xi_t^{i,j}}{(\sigma_t^{i,j})}\|^2 - \|\frac{\xi_{t-1}^{i,j}}{(\sigma_{t-1}^{i,j})}\|^2\right)^2 \le$$
$$\le \frac{K}{2n}\sum_{i\in M_t^j}\left(\frac{M_4}{(\sigma_t^{i,j})^4}+\frac{M_4}{(\sigma_{t-1}^{i,j})^4}-2\right) \le c_4^j.$$

Since we consider that $v_t = 0$ then in Assumption 6 $c_v = 0$. Assumption 7 is satisfied because of the noise independence in the observation model.

The proof of Theorem 2 is now complete.