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Sign-Perturbed Sums Approach for Data Treatment of Dynamic Fracture Tests

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Abstract— **The Sign-Perturbed Sums (SPS) randomized algorithm is adapted to the procedure of data treatment of dynamic fracture tests. The original method is modified and applied for nonlinear regression function describing the strain rate dependence of material strength at the framework of structure-temporal approach. Ordinary there are few observation points of goal parameter with random noises with unknown statistical distribution, hence this stipulates the choice of SPS-algorithm for this problem. It is proved that SPS-procedure permits to define confidence intervals for dynamic strength parameter τ with proper accuracy in this case. The applicability of this method is demonstrated on example of experimental data treatment of dynamic fracture of a concrete.**

I. Introduction

The problem of observations with following estimation of parameter values is typical procedure in modern engineering. If we concern direct measurements obtained by uncomplicated way then there are common methods allow us to determine some interval of values for the parameter with certain degree of an accuracy. These methods based on the assumption of the normal distribution of random noises and they demand a large number of data points, for example least meant squares (LMS) method. The main peculiarity of such methods is absolute confidence of obtained intervals, that is no necessary in most practical cases where it is enough to know that one or another parameter takes up the value from certain interval with sufficiently great probability. Additionally there is often no large quantity of data points for the analysis and there is no information about noises. Thus, we should apply some other methods in these cases which are capable to make some estimation of parameter value. One of these methods is SPS-procedure [1], which allows us to determine the confidence interval under very weak assumptions about the random noise and relatively small quantity of data points.

The measuring of material strength and rheological parameters is one of the main problems in mechanical engineering. The tests to measure static strength or Young elastic modulus have been normalized during the time and are performed by standard direct stress machine. These experiments are not labor consuming and provide a lot data points for analysis. Thus, traditional methods like LMS permit to obtain an average value of required parameter with certain degree of an accuracy, for example $\sigma_c \in [\sigma_{c-}; \sigma_c{}^+]$. Also, direct observations stipulate that the physical sense of the measured parameter is established and well-known, and all researchers have unified conception about final result.

The more complicated situation is observed in dynamics, where material strength cannot be characterized by one parameter of the critical stress. Experimental test results demonstrate that under intensive impacts, specimens can resist to stress level significantly higher than their static strength σ_c and stress value at fracture moment σ_* depends on rate of loading and a shape of breaking pulse [2–9]. There is a common point of view that in dynamics fracture occurs at the time moment when breaking stresses reach the maximum like in statics, then the value of σ_* could be noted as material dynamic strength σ_d depending on the strain-rate of load. These strain-rate dependencies are interpreted as passport specifications of materials in certain approaches. However, the variety of strain-rate curves is infinite due to strong influence of impact condition to σ_d value, and it is impossible to describe material strength by the same way when fracture is initiated by threshold impacts. In these cases stress level at the breaking moment can be less even the static strength, this phenomenon is called the fracture delay effect.

The peculiarities of dynamic fracture mentioned above served to development of essential other ways to describe the dynamic strength of materials. One of them is the structure-temporal approach based on the incubation time criterion of fracture [10, 11]. The main idea of this criterion is that fracture does not occur instantly and every transient process has own characteristic time. The addition of only one parameter - incubation time τ allows us to predict stress level at the fracture moment and calculate strain-rate dependencies for all type of impacts. This approach was successfully applied for many different dynamic problems such as predicting of dynamic strength for different materials and condensed matters e.g. dynamic fracture of rocks and concretes, dynamic yielding of metals, acoustic ultrasonic cavitation of liquids etc [11–14]. Thus, the incubation time τ in couple of static critical stress σ_c is completely determine the dynamic strength of material at the framework of structure-temporal approach.

All dynamic tests are more elaborate and labor consuming than static ones, moreover, tests with direct measuring of incubation time are very complicated to be realized. Thus, the value of τ could be obtained implicitly by ensuring good correspondence between a model theoretical curve and

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a scatter of the experimental data. Any method could be applied for fitting of the model curve, e.g. LMS-method, but it gives only one value of the incubation time and there is no any estimation of inaccuracy when we have the lack of experimental data productive of an absence of any adequate variability in observations. It should be repeated that standard estimation algorithms use typically the condition of persistent excitation in data. However, this condition is difficult to ensure in the considered problem due to complexity of dynamic tests and hence low quantity of experimental data points. This often results in a degenerate observation data and complicated identification problems.

The well-known *set-membership approach* to identification uses no statistical properties of the noise but assume some known upper bounds on uncertain system components instead. The purpose of the approach is typically to compute some upper or lower set estimates on the set of data-consistent parameters and no convergence of this set estimates to the true unknown parameter vector can be achieved without any significant additional assumptions (see, e.g. [15, 16]). In the context of considered problem, the using of robust estimates of minimax methods gives a very wide interval for τ, with a high inaccuracy and this result is very conservative. The boundary values of this minimax interval of τ correspond to model curves passed through low and high points of the experimental data.

New SPS-procedure suggested in [1] provide rigorously guaranteed non-asymptotic confidence interval for the unknown parameters of a linear dynamical control plant in the small-sample setting. We adopt this approach to the problem of evaluation of the incubation time τ for nonlinear regression function. This way is proper to this problem, because it can to provide a confidence region of τ with admissible for engineering rate of accuracy. Approximately ten data points are enough to determine an average value of incubation time with the accuracy $\epsilon = 20 - 35\%$. The applicability of the new algorithm to the incubation time approach will be illustrated on few dynamic fracture tests of concrete [17].

The paper is organized as follows. Section II describes main aspects of the structural-temporal approach and ways of incubation time criterion application to prediction of dynamic material strength. In Section III, we introduce a formal problem setting. Section IV introduces the main assumptions and describes a special SPS method. The illustrative experiments examples are presented in Section V. Section VI concludes the paper.

II. Incubation Time Approach

The general form of the incubation time criterion of fracture is:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\left(\frac{\sigma(t')}{\sigma_c}\right)^{\alpha} dt' \leq 1 \tag{1}$$

where $\sigma(t')$ describes to loading stress, τ is incubation time of fracture, α is dimensionless parameter, for most brittle materials $\alpha = 1$. According to (1), fracture does not occur while left part of criterion is less than one. The fracture moment t_* corresponds to the time value t when the left part of criterion for the first time become to equal to one.

The linear growth of stresses until the fracture is realized in most experimental schemes in dynamic fracture tests. Thus, the impact shape function can be determined by strain-rate of load $\dot{\epsilon}$ and elastic modulus k:

$$\sigma(t) = h(t)k\dot{\epsilon}t \tag{2}$$

where $h(t)$ is Heaviside step function. Substitution the function (2) to criterion (1) leads to follow equation for fracture time t_*:

$$h(t_*)\left(\frac{t_*}{\tau}\right)^{\alpha+1} - h(t_*-\tau)\left(\frac{t_*}{\tau}-1\right)^{\alpha+1} = s \tag{3}$$

where $s = (\alpha+1)(\sigma_c/(k\dot{\epsilon}\tau))^{\alpha}$ is dimensionless parameter, which value depends on strain-rate of load impacts. Fracture time t_* could not be negative, hence $h(t_*) = 1$, and there are two eventualities when $t_* > \tau$ or $t_* \leq \tau$. From expression (3) it follows, that $s = 1$ for $t_* = \tau$, hence final equation set determining fracture moment t_* is:

$$\begin{cases} (t_*/\tau)^{\alpha+1} = s, & s < 1, \\ (t_*/\tau)^{\alpha+1} - (t_*/\tau - 1)^{\alpha+1} = s, & s \geq 1. \end{cases} \tag{4}$$

Thus, we obtain the strain-rate dependence of the dynamic threshold of fracture by substitution the solution of (4) to shape function of load stresses (2) $\sigma_*(\dot{\epsilon}) = k\dot{\epsilon}t_*$. Let us assume the parameter $\alpha = 1$ since this value corresponds to dynamic behavior of the concrete in experimental tests which hereafter are analyzed. Then roots of the equation (4) could be expressed in an explicit form and critical fracture stress is follow:

$$\sigma_*(\dot{\epsilon}) = \varphi(\tau,\dot{\epsilon}) = \begin{cases} \sigma_c + \frac{\tau}{2}k\dot{\epsilon}, & \dot{\epsilon} \leq \frac{2\sigma_c}{k\tau}, \\ \sqrt{2\sigma_c\tau k\dot{\epsilon}}, & \dot{\epsilon} > \frac{2\sigma_c}{k\tau}. \end{cases} \tag{5}$$

The application of the incubation time criterion is demonstrated on example of impact compressive fracture test on the concrete [17]. The theoretical curve in comparison with experimental points is shown in Fig.1. The value of $\tau = 10.02\mu s$ (full line) is calculated by LMS-method, as it is mentioned above this result does not provide any information about inaccuracy of this value. Another minimax method gives very wide interval for incubation time value $\tau \in [7.52; 15.02]\mu s$, and does not provide any information about properties, that new experimental points would lie in this interval. Hence, we apply new SPS-method of data treatment, which gives, at first, more accurate interval for possible values of τ and, at second, degree of its confidence in a proper range.

III. Problem Description

We can choose $\dot{\epsilon}$ as an acting factor in dynamic test experiments and get the correspondence to observation σ_*. Dynamic test data are satisfied to the following model of N observations with noise:

$$\sigma_{*i} = \varphi(\tau,\dot{\epsilon}_i) + v_i,\ i = 1, 2, \ldots, N \tag{6}$$

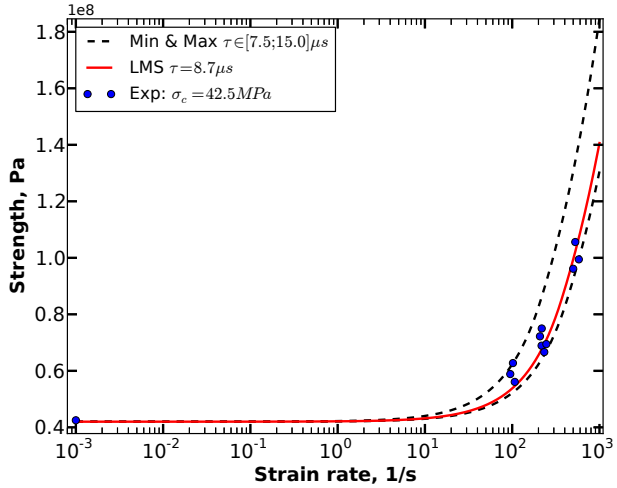

Fig. 1: Dynamic fracture of the concrete [17]. Blue points are experimental data. Lines are theoretical curves plotted by incubation time fracture criterion: full line - least mean squares method; dash line - min & max robust method

where v_i is a independent random noise (an inaccuracy) with symmetrical distribution. If we assume that the strain-rate dependence is obeyed to principles of structure-temporal approach, then function:

$$\varphi(\tau, \dot{\epsilon}_i) = k\dot{\epsilon}t_*(\tau) \tag{7}$$

where t_* is fracture moment predicted by incubation time criterion by solving the equation (4).

The equation (7) permits to calculate fracture stress for different τ, then the least mean square method (LMS) gives the best fitted value of the incubation time, for that follow sum has a minimum value:

$$\sum_{i=1}^{N}(\varphi(\tau, \dot{\epsilon}_i) - \sigma_{*i})^2 \to \min_{\tau}. \tag{8}$$

However, we do not able to get a sufficiently good confidence interval for unknown τ without significant restrictions for the noise v_i when N is small.

Objective: Our goal is to construct confidence regions for unknown τ that have guaranteed user-chosen confidence probabilities for finite, and possibly small, number of data points. *It must be defined* by the observations of outputs $\{\sigma_{*i}\}_{i=1}^N$ and known acting factors $\{\dot{\epsilon}_i\}_{i=1}^N$ which may be chosen. The constructed regions are quasi distribution-free, as the only assumption on the noise has a property of symmetry. This is important since in practice the knowledge about the noise distribution is limited. Additionally, the confidence regions should contain the least-squares point estimate.

IV. SPS Procedure for Constructing of Confidence Regions

For a finite number of observations we can use the following procedure which is similar to SPS procedure from [1]. The LMS estimate is obtained as the solution of the equation

$$H_0(\tau) = \sum_{i=1}^{N}(\sigma_{*i} - \varphi(\tau, \dot{\epsilon}_i))\frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau} = 0 \tag{9}$$

where

$$\frac{d\varphi(\tau, \dot{\epsilon})}{d\tau} = \begin{cases} \frac{1}{2}k\dot{\epsilon}, & \dot{\epsilon} \le \frac{2\sigma_c}{k\tau}, \\ \frac{1}{\sqrt{2\tau}}\sqrt{\sigma_c k\dot{\epsilon}}, & \dot{\epsilon} > \frac{2\sigma_c}{k\tau}. \end{cases} \tag{10}$$

We will try to exploit the information in the data as much as possible while assuming minimal prior statistical knowledge about the noise. Our core assumption is the symmetry of the noise. For some $M > 0$ we generate $M \times N$ Bernoulli random values $\beta_{ij} = \pm 1$ with probability $\frac{1}{2}$, and introduce $M - 1$ sign-perturbed sums

$$H_j(\tau) = \sum_{i=1}^{N}\beta_{ij}(\sigma_{*i} - \varphi(\tau, \dot{\epsilon}_i))\frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau}, \tag{11}$$

$j = 1, 2, \ldots, m - 1$.

If $\tau^\star$ is a nominal value of τ then $H_0(\tau^\star)$ and $H_j(\tau^\star)$ have the same distribution since $\{v_i\}$ are symmetric. Therefore, there is no reason why a particular $|H_j(\tau^\star)|$ should be bigger or smaller than another $|H_{j'}(\tau^\star)|$ and the probability that a particular $|H_j(\tau^\star)|$ is the m-th largest one in the ordering of $\{|H_j(\tau^\star)|\}_{j=0}^{M-1}$ will be the same for all j, including $j = 0$ (the case where there are no sign-perturbations). As can take on different values, this probability is exactly $\frac{1}{M}$.

Algorithm:

1) Given a (rational) confidence probability $p \in (0, 1)$, set integers $M > q > 0$ such that $p = 1 - q/M$.
2) Generate $N(M - 1)$ i.i.d. random signs $\{\beta_{ij}\}$ with $\text{Prob}\{\beta_{ij} = 1\} = \text{Prob}\{\beta_{ij} = -1\} = \frac{1}{2}$ for $i \in \{1, 2, \ldots, N\}$ and $j \in \{1, 2, \ldots, M - 1\}$.
3) Set

$$\widehat{\mathcal{T}} := \{\tau : SPS\_Indicator(\tau) == 1\}.$$

Procedure: $SPS\_Indicator(\tau)$

1) For the given τ compute the prediction error for $i \in \{1, 2, \ldots, N\}$

$$\delta_i(\tau) = \sigma_{*i} - \varphi(\tau, \dot{\epsilon}_i).$$

2) Evaluate

$$\begin{aligned} H_0(\tau) &= \sum_{i=1}^{N}\delta_i(\tau)\frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau}, \\ H_j(\tau) &= \sum_{i=1}^{N}\beta_{ij}\delta_i(\tau)\frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau}, \end{aligned}$$

for $j \in \{1, 2, \ldots, M - 1\}$.

3) Order scalars $|H_j(\tau)|$ from smallest to biggest.
4) Compute the rank $\mathcal{R}(\tau)$ of $|H_0(\tau)|$ in the ordering, where $\mathcal{R}(\tau) = 1$ if $|H_0(\tau)|$ is the smallest in the ordering, $\mathcal{R}(\tau) = 2$ if $|H_0(\tau)|$ is the second smallest, and so on.
5) Return 1 if $\mathcal{R}(\tau) \le M - q$, otherwise Return 0.

Note that the LMS estimate $\hat{\tau}$ has by definition the property that $H_0(\hat{\tau}) = 0$. Therefore, the LMS estimate $\hat{\tau}$ is included in the SPS confidence region.

The probability that $\tau^\star$ belongs to $\widehat{\mathcal{T}}$ is given in the following Theorem.

Theorem 1. **If** the noise has a property of symmetry **then**

$$\text{Prob}\{\tau^\star \in \widehat{\mathcal{T}}\} = 1 - \frac{q}{M} \tag{12}$$

where M, q, $\widehat{\mathcal{T}}$ from Steps 1 and 3 of the algorithm described above.

Proof: The function $\varphi(\tau, \dot{\epsilon})$ is convex continuous and monotonically increasing with growth of parametric variable τ since the derivative $d\varphi(\tau)/d\tau$ is positive for every $\tau > 0$. Also, the critical strain-rate $\dot{\epsilon}_c = 2\sigma_c/k\tau$ decreasing with growth of τ, hence $\varphi(\tau_1, \dot{\epsilon}) > \varphi(\tau_2, \dot{\epsilon})$ for every $\dot{\epsilon}$ under condition that $\tau_1 > \tau_2$. Therefore, if τ is not among the interval obtained by minimax method - $\tau \notin [\tau_{min}; \tau_{max}]$, then modeling curves lie higher or below of all data points on the plot (Fig.1).

Let us consider the case $\tau < \tau_{min}$ when all predicted values of the strength are less the experimental ones: $\varphi(\tau, \dot{\epsilon}) < \sigma_{*i}, \forall i = 1, ..., N$. Then all terms of the sum $H_0(\tau)$ in (9) are positive and this sum is greater then every other sum $H_j(\tau)$, since some terms of the last one are negative due to random signs β_{ij}, i.e. $|H_0(\tau)| > |H_j(\tau)|, \forall \tau < \tau_{min}$ and $j = 1, ..., M-1$. Therefore, $\mathcal{R}(\tau) = M$ and all values of $\tau < \tau_{min}$ are rejected by $SPS\_Indicator(\tau)$ procedure. Analogous reasoning leads to all values of $\tau > \tau_{max}$ are also separated. All terms of the sum $H_0(\tau)$ are negative and $|H_0(\tau)| > |H_j(\tau)|$, since $H_j(\tau)$ contains certain positive terms. Thus, the minimax region contains the interval $\widehat{\mathcal{T}}$ derived by SPS-method.

It should be noted that the $\delta_i(\tau) = v_i$ for the nominal value of τ. Then applying Lemma 1 from [1], it is follows that $H_j(\tau)$ are i.i.d, since β_{ij} are i.i.d signs and $\delta_i(\tau)$ are symmetric. Thus, the probability that some $H_j(\tau)$ takes a certain place in the ordering $\{|H_j(\tau)|\}_{j=0}^{M-1}$ is equal to $1/M$ for $\forall \tau \in \widehat{\mathcal{T}}$. Then Theorem 1 is proved. □

V. Experiments

Results of SPS-procedure application to experimental data of dynamic fracture tests of the concrete are shown in Fig.2. There are two experimental series for concretes with a little different in values of the static strength $\sigma_c^{(1)} = 42.5MPa$ and $\sigma_c^{(2)} = 40.8MPa$. All calculations were performed for the first series of data points, the second one is used to check of the obtained results.

These values of confidence probabilities 90% and 99.8% were achieved by follows values of SPS-procedure parameters $M = 200, q = 20$ and $M = 500, q = 1$. This gives us two intervals of incubation fracture time $\tau_{0.9} \in [8.0; 10.4]\mu s$ and $\tau_{0.998} \in [7.4; 14.7]\mu s$, while LMS-method provides $\tau = 8.7\mu s$. We can see that 99.8% confidence interval approximately coincides with the minimax estimation. It is the direct consequence from the Theorem. Thus, SPS-procedure allow us to calculate incubation time value with relatively small inaccuracy 20% with quite big degree of confidence 90%. Also, the orange checking points of the seconds series lie into wide 99.8% confidence region, while only three of them are almost in the 90% interval.

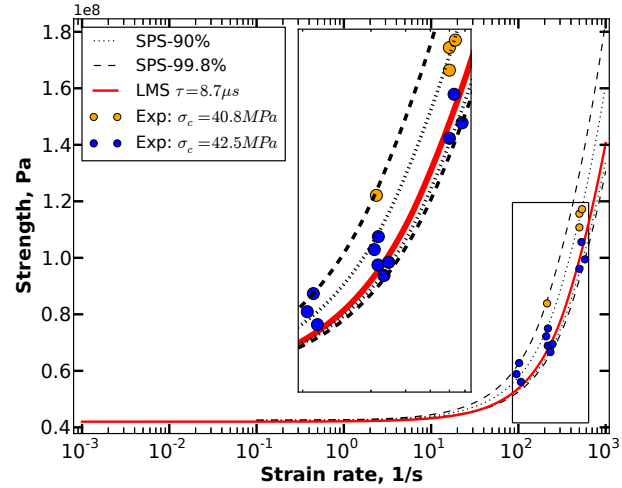

Fig. 2: Dynamic fracture of concrete [17]: blue circles - $\sigma_c^{(1)} = 42.5MPa$; orange circles $\sigma_c^{(2)} = 40.8MPa$. Red line is plotted by LMS-method $\tau = \mu s$; dotted lines are plotted by SPS-procedure for $\tau_{0.90} \in [8.0; 10.4]\mu s$; Dashed lines - $\tau_{0.998} \in [7.4; 14.7]\mu s$

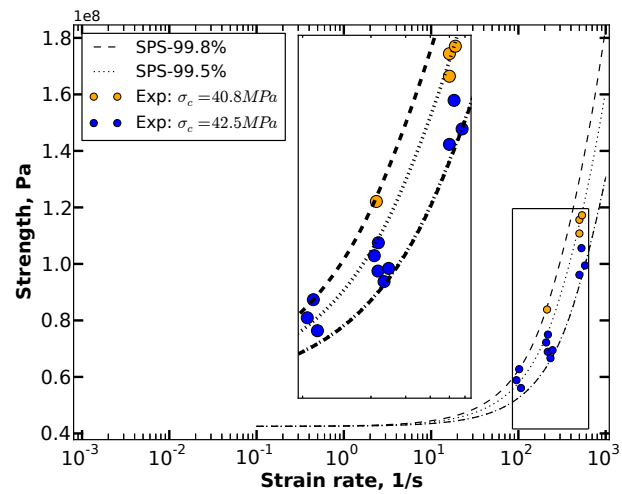

Fig. 3: Dynamic fracture of concrete [17]: blue circles - $\sigma_c^{(1)} = 42.5MPa$; orange circles $\sigma_c^{(2)} = 40.8MPa$. Dotted lines are plotted by SPS-procedure for $\tau_{0.995} \in [7.5; 11.3]\mu s$; Dashed lines - $\tau_{0.998} \in [7.4; 14.7]\mu s$

The interesting result is obtained for the follow parameters of the SPS-procedure $M = 200, q = 1$ when we calculate the 99.5% confidence region (Fig.3). In spite of approximately the same confidential probability 99.8% and 99.5% the second interval is significantly narrower than the first one, but its rate of accuracy is not greater than 30%.

VI. Conclusions

The main problem in dynamic fracture mechanics is absence of standard methods to compute and predict the limit characteristics of intensive impacts for different materials and condensed matters. The incubation time approach proved itself as good instrument for solution of mentioned problems, but it also has not standard procedure to analyze experimental data for estimation of the incubation time value. The main difficulty is that incubation time τ could be measured only implicitly by performing complicated dynamic test, therefore standard measuring methods based on the law of a large numbers do not work. The proposed application of SPS-procedure shows how to calculate incubation time with given proper inaccuracy under limit quantity of experimental points. This SPS-algorithm also permits to estimate the confidence of obtained intervals for parameter τ that it is very important for applications.

The further researches are intended to make more weak assumptions on random noises like in works [18–20] and to perform real experimental test with the given by adaptive approach values of the acting factor $\dot{\epsilon}$ in accordance to principles presented in [21]. Also we want to consider materials which behavior corresponded to case $\alpha > 1$. There is no explicit form of solution in this case, but the main peculiarities of the nonlinear regression function would be the same.

References

[1] B. Csaji, M. C. Campi, , and E. Weyer, "Sign-perturbed sums: A new system identification approach for contructing exact non-asymptotic confidence regions in linear regression models," *IEEE Trans. on Signal Processing*, vol. 63, no. 1, pp. 169–181, 2015.

[2] J. D. Campbell and W. G. Ferguson, "The temperature and strain-rate dependence of shear strength of mild steel," *The Philosophical Magazine*, vol. 21, pp. 63–82, 1970.

[3] A. J. Rosakis, J. Duffy, and L. B. Freund, "The determination of dynamic fracture toughness of alsi 4340 steel by the shadow spot method," *J. Mech. Phys. Solids*, vol. 32, no. 4, pp. 443–160, 1984.

[4] D. A. Shockey, L. Seaman, and D. R. Curran, *Material behavior under high stress and ultrahigh loading rates*. Springer US, New York, 1983.

[5] G. R. Johnson and W. H. Cook, "Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures," *Engineering Fracture Mechanics*, vol. 21, pp. 31–48, 1985.

[6] V. S. Nikiforovsky and E. I. Shemyakin, *Material behavior under high stress and ultrahigh loading rates*. Novosibirsk,[in Russian], 1979.

[7] K. Ravi-Chandar, "Experimental challenges in the investigation of dynamic fracture of brittle materials," *Physical Aspects of Fracture. NATO Science Series, Ser. II : Mathematics, Physics and Chemistry*, vol. 32, pp. 323–342, 2001.

[8] J. F. Kalthoff and S. Wincler, "Failure mode transition at high rates of shear loading," vol. May, 1987, pp. 161–176.

[9] G. I. Kanel, S. V. Razorenov, K. Baumung, and J. Singer, "Dynamic yield and tensile strength of aluminum single crystals at temperatures up to the melting point," *Journal of Applied Physics*, vol. 90, no. 1, pp. 136–143, 2001.

[10] Y. V. Petrov and A. A. Utkin, "Dependence of the dynamic strength on loading rate," *Mater. Science*, vol. 25, no. 2, pp. 153–156, 1989.

[11] Y. V. Petrov, "Incubation time criterion and the pulsed strength of continua: Fracture, cavitation, and electrical breakdown," *Doklady Physics*, vol. 49, no. 4, pp. 246–249, 2004.

[12] A. A. Gruzdkov and Y. V. Petrov, "Cavitation breakup of low- and high-viscosity liquids," *Technical Physics*, vol. 53, no. 3, pp. 291–295, 2008.

[13] N. M. Pugno, "Dynamic quantized fracture mechanics," *International Journal of Fracture*, vol. 140, no. 1-4, pp. 159–168, 2006.

[14] Q. Z. Wang, S. Zhang, and H. P. Xie, "Rock dynamic fracture toughness tested with holed-cracked flattened brazilian discs diametrically impacted by shpb and its size effect," *Experimental Mechanics*, vol. 50, pp. 877–885, 2010.

[15] E.-W. Bai, K. M. Nagpal, and R. Tempo, "Bounded-error parameter estimation: Noise models and recursive algorithms," *Automatica*, vol. 32, no. 7, pp. 985–999, 1996.

[16] F. Blanchini and M. Sznaier, "A convex optimization approach to synthesizing bounded complexity h_{inf} filters," *IEEE Transactions on Automatic Control*, vol. 57, no. 1, pp. 219–224, 2012.

[17] L. Yong and X. Kai, "Modelling of dynamic behaviour of concrete materials under blast loading," *International Journal of Solids and Structures*, vol. 41, pp. 131–143, 2004.

[18] K. Amelin, N. Amelina, O. Granichin, and O. Granichina, "Combined procedure with randomized controls for the parameters' confidence region of linear plant under external arbitrary noise," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*. IEEE, 2012, pp. 2134–2139.

[19] K. Amelin and O. N. Granichin, "Randomized control strategies under arbitrary external noise," *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1328–1333, 2016.

[20] A. Senov, K. Amelin, N. Amelina, and O. N. Granichin, "Exact confidence regions for linear regression parameters under external arbitrary noise," *Proc. of the 2014 American Control Conference (ACC) Portland, USA, 4-6 June*, pp. 5097–5102, 2014.

[21] O. N. Granichin, "The nonasymptotic confidence set for parameters of a linear control object under an arbitrary external disturbance," *Automation and Remote Control*, vol. 73, no. 1, pp. 20–30, 2012.