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Using Stochastic Approximation Type Algorithm for Choice of Consensus Protocol Step-Size in Changing Conditions ⋆

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Abstract: In the paper a multi-agent network system of different computing nodes is considered. A problem of load balancing in the network is addressed. The problem is formulated as consensus achievement problem and solved via local voting protocol. Agents exchange information about their states in presence of noise in communication channels. At certain moment network system topology changes and new step size of control protocol is chosen to meet new conditions. Step size adjustment is done by stochastic approximation type algorithm. Analytically obtained optimal step size values are given. Simulation example demonstrating step size adjustment is provided.

Keywords: load balancing, consensus achievement, stochastic approximation, multi-agent networks

1. INTRODUCTION

Important practical problem in network systems is problem of load balancing. It may arise in such network systems as computer, production, transport, logistics, and other service networks. In computational networks load balancing is needed to improve system efficiency. A multi-agent approach is used to address this problem in network systems. A possible goal for control in such systems is to improve the network speed of operation using communication among agents in the system. In Amelina et al. (2015a) it was shown that the problem of almost optimal task distribution among agents could be reformulated as a problem of the consensus achievement in the network.

The consensus approach was widely applied for solving various practical problems such as cooperative control of multivehicle networks Ren et al. (2007); Granichin et al. (2012), distributed control of robotic networks Bullo et al. (2009), flocking problem Yu et al. (2010a); Virágh et al. (2014), optimal control of sensor networks Kar and Moura (2010) and others. Works Ren and Beard (2007); Chebotarev and Agaev (2009); Li and Zhang (2009); Yu et al. (2010b); Huang (2012); Proskurnikov (2013); Lewis et al. (2014) consider formulating the conditions for achieving consensus in such systems.

In Amelina et al. (2015b) a choice of an optimal step-size of consensus-type protocol for task redistribution among agents in a stochastic network with randomized priorities is considered. It is shown that a trade-off is made between noise sensitivity and the rate of convergence of control protocol while choosing its step-size. The paper proposes a way of choosing step-size to maximize convergence precision.

An optimal step-size of control protocol could be chosen for the network system under certain conditions such as parameters of noise during information exchange, delays occurring in communication channels, system topology etc. But the value of optimal step-size corresponding to different conditions of system operation might be different. In Granichin and Amelina (2015) a problem of tracking under influence of disturbances via simultaneous perturbation stochastic approximation is considered. The paper addresses the problem in general formulation with little assumption about disturbances.

Simultaneous perturbation stochastic approximation (SPSA) was proposed by Spall Spall (1992) and can be used for solving optimization problems in case when it is difficult or impossible to obtain a gradient of the objective function with respect to the parameters being optimized.

In this paper we propose a way to adjust step-size parameter of control strategy in changing conditions. We use a stochastic approximation type procedure to update values of step-size during multi-agent system operation. Obtained analytically optimal step-sizes for the proposed control protocol are provided.

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The paper is organized as follows. Notation used in the paper and the problem formulation are given in Section II. The control protocol for achieving the consensus is introduced in Section III. In Section IV the main assumptions and main results are presented. Simulation results are given in Section V. Section VI contains conclusion remarks.

2. PROBLEM STATEMENT

Let's consider a dynamic network system of n agents, which exchange information among themselves during tasks processing. Tasks may come to different agents of the system in different discrete time instants $t = 1,\ldots$. Agents process incoming tasks in parallel. Tasks can be redistributed among agents based on a feedback.

Without loss of generality, agents in the system are numbered. Assume $N = \{1,\ldots,n\}$ denotes the set of agents in the network system. Let $i \in N$ be the number of an agent. The network topology may switch over time. Let the dynamic network topology be modeled by a sequence of digraphs $\{(N,\mathscr{E}_t)\}_{t\geq 0}$, where $\mathscr{E}_t \subset \mathscr{E}$ denotes the set of edges at time t of topology graph $(N,\mathscr{E}_t)$. The corresponding adjacency matrices are denoted as $A_t = [a_t^{i,j}]$, where $a_t^{i,j} > 0$ if agent j is connected with agent i and $a_t^{i,j} = 0$ otherwise. Here and below, an upper index of agent i is used as the corresponding number of an agent (not as an exponent). Denote $\mathscr{G}_{A_t}$ as the corresponding graph.

To introduce some properties of the network topology, the following definitions from the graph theory will be used. Define *the weighted in-degree* of node i as the sum of i-th row of matrix A: $indeg^i(A) = \sum_{j=1}^n a^{i,j}$; $\mathscr{D}(A) = \mathrm{diag}\{indeg^i(A)\}$ is the corresponding diagonal matrix; $indeg_{\max}(A)$ is the maximum in-degree of graph $\mathscr{G}_A$. Let $\mathscr{L}(A) = \mathscr{D}(A) - A$ denote the *Laplacian* of graph $\mathscr{G}_A$; $\cdot^{\mathrm{T}}$ is a vector or matrix transpose operation; $||A||$ is the Euclidian norm: $||A|| = \sqrt{\sum_i \sum_j (a^{i,j})^2}$; $Re(\lambda_2(A))$ is the real part of the second eigenvalue of matrix A ordered by the absolute magnitude; $\lambda_{\max}(A)$ is the maximum eigenvalue of matrix A.

It is said that digraph $\mathscr{G}_B$ is a subgraph of a digraph $\mathscr{G}_A$ if $b^{i,j} \leq a^{i,j}$ for all $i,j \in N$.

Digraph $\mathscr{G}_A$ is said to contain a *spanning tree* if there exists a directed tree $\mathscr{G}_{tr} = (N,\mathscr{E}_{tr})$ as a subgraph of $\mathscr{G}_A$ which includes all vertices of $\mathscr{G}_A$.

The behavior of agent $i \in N$ is described by characteristics of two types:

- lengths of queue of tasks at time instant t: q_t^i,
- productivity: p^i.

Let random variable η_j denote complexity (or a number of computational operation needed to execute the task) of a task which came to the system. Dynamics of the system can be written in the following way:

$$\sum_{q_{t+1}^i} \eta_j = \sum_{q_t^i} \eta_{j'} - p^i + \sum_{z_t^i} \eta_{j''} + \sum_{u_t^i} \eta_{j'''},$$

where $\sum_{q_{t+1}^i} \eta_j$ is number of computational operations needed to execute all tasks in the queue of agent i at time instant $t+1$, p^i is productivity of agent i or the number of computational operations it can perform during one tact of the system (assume it is constant), $\sum_{z_t^i} \eta_{j''}$ is the complexity of tasks which came to the system on agent i at time instant t and $\sum_{u_t^i} \eta_{j'''}$ is the complexity of tasks which already came to other agents at previous time instants and were redistributed to agent i according to control protocol.

Assume random variable η has mathematical expectation $\bar{\eta} < \infty$. Let's take expectation of left and right parts of the equation of system dynamics.

$$E\left(\sum_{q_{t+1}^i} \eta_j\right) = E\left(\sum_{q_t^i} \eta_{j'} - p^i + \sum_{z_t^i} \eta_{j''} + \sum_{u_t^i} \eta_{j'''}\right)$$

$$\sum_{q_{t+1}^i} \bar{\eta} = \sum_{q_t^i} \bar{\eta} - p^i + \sum_{z_t^i} \bar{\eta} + \sum_{u_t^i} \bar{\eta}$$

Left and right parts are now equal to number of tasks at agent i multiplied by their average complexity.

$$\bar{\eta} \sum_{q_{t+1}^i} 1 = \bar{\eta} \sum_{q_t^i} 1 - p^i + \bar{\eta} \sum_{z_t^i} 1 + \bar{\eta} \sum_{u_t^i} 1$$

$$\bar{\eta} q_{t+1}^i = \bar{\eta} q_t^i - p^i + \bar{\eta} z_t^i + \bar{\eta} u_t^i$$

Divide both parts of the equation by constant value $\bar{\eta}$. We get discrete model which allows as to analyze system dynamics without information about complexities of each task in the system (but with assumption their average value is bounded). For all $i \in N$, $t = 0,1,\ldots$, the dynamics of the network system in a vector form is as follows

$$q_{t+1}^i = q_t^i - \tilde{p}^i + z_t^i + u_t^i, \tag{1}$$

where $\tilde{p}^i = p^i/\bar{\eta}$, z_t^i the amount of new tasks, which came to the system and were received by agent i at time instant t; u_t^i is control action (redistributed tasks to agent i at time instant t), which is chosen based on some information about queue lengths of neighbors q_t^j, $j \in N_t^i$, where N_t^i is the set $\{j \in N : a_t^{i,j} > 0\}$.

Denote

$$x_t^i = \frac{q_t^i}{\tilde{p}^i} \tag{2}$$

the *load* of agent $i \in N$. Assume, that $\tilde{p}^i \neq 0$, $\forall i \in N$. In Amelina et al. (2015a) it was proven that from all possible options for the redistribution of all tasks the minimum operation time of the system is achieved when loads x_t^i are equalized throughout the network. Hence, it is important to consider the achievement of the following goal.

It is required to maintain balanced (equal) loads across the network under conditions of changing network topology.

At this setting we can consider the consensus problem for states x_t^i of agents, where x_t^i is a state of agent $i \in N$. We use the following definitions.

Definition *1.* n agents of a network are said to reach a *consensus* at time t if $x_t^i = x_t^j \ \forall i,j \in N, i \neq j$.

Definition *2.* n agents are said to achieve *asymptotic mean square ε-consensus* for $\varepsilon > 0$ when

$$\overline{\lim}_{t\to\infty} \mathrm{E}\|x_t^i - x_t^j\|^2 \leq \varepsilon.$$

To ensure balanced loads across the network (e.g., in order to increase the overall throughput of the system and to reduce the execution time), it is naturally to use a redistribution protocol over time. We assume that to form the control (redistribution) strategy each agent $i \in N$ has noisy observations about its neighbors' states

$$y_t^{i,j} = x_t^j + w_t^{i,j},\ j \in N_t^i, \tag{3}$$

where $w_t^{i,j}$ is a noise occurring during transmission from node j to node i.

3. CONTROL PROTOCOL

In Amelina et al. (2015a), properties of a control algorithm, called local voting protocol, were studied for stochastic networks in the context of load balancing problem. For each agent the control (amount of redistributed tasks) was determined by the weighted sum of differences between the information about the state of the agent and the information about its neighbors' states. Let's consider a protocol as follows. We define

$$u_t^i = \gamma \tilde{p}_t^i \sum_{j \in \bar{N}_t^i} b_t^{i,j}(y_t^{i,j} - x_t^i), \tag{4}$$

where $\gamma > 0$ is a step-size of the control protocol and $\bar{N}_t^i \subset N_t^i$ is the neighbor set of agent i (note, that we could use not all the available connections, but some subset of them), $b_t^{i,j}$ are protocol coefficients.

Let $B_t = [b_t^{i,j}]$ be the matrices of task redistribution protocol for every time instant t. (We set $b_t^{i,j} = 0$ when $a_t^{i,j} = 0$ or $j \notin \bar{N}_t^i$.) The corresponding graph $\mathscr{G}_{B_t}$ may have the same topology as graph $\mathscr{G}_{A_t}$ of matrix A_t or more poor.

The dynamics of the closed loop system with protocol (4) will be as follows:

$$x_{t+1}^i = x_t^i - 1 + \tilde{z}_t^i + \gamma \sum_{j \in \bar{N}_t^i} b_t^{i,j}(y_t^{i,j} - x_t^i) =$$

$$x_t^i - 1 + \tilde{z}_t^i + \gamma \left(\sum_{j \in \bar{N}_t^i} b_t^{i,j} x_t^j \right) - \gamma indeg^i(B_t) x_t^i + \gamma \tilde{w}_t^i,\ i \in N, \tag{5}$$

where $\tilde{w}_t^i = \sum_{j \in \bar{N}_t^i} b_t^{i,j} w_t^{i,j}$ and $\tilde{z}_t^i = z_t^i / \tilde{p}^i$.

Let us rewrite Eq. (5) in a more compact form. Define the $\mathbb{R}^n$-valued vectors $X_t = [x_t^i]$, $\mathbf{1}_n$ - vector with all elements equal to 1, $Z_t = [\tilde{z}_t^i]$ and $W_t = [\sum_{j \in \bar{N}_t^i} b_t^{i,j} w_t^{i,j}]$. The dynamics of the closed loop system with protocol (4) may be represented as

$$X_{t+1} = X_t + \gamma(B_t - \mathscr{D}(B_t))X_t - \mathbf{1}_n + Z_t + \gamma W_t. \tag{6}$$

Due to the view of Laplacian matrices $\mathscr{L}(B_t)$ we can rewrite the dynamics of the system in the following vector-matrix form:

$$X_{t+1} = X_t - \gamma \mathscr{L}(B_t) X_t - \mathbf{1}_n + Z_t + \gamma W_t. \tag{7}$$

4. MAIN RESULTS

4.1 Assumptions

Let $(\Omega, \mathscr{F}, P)$ be the underlying probability space corresponding to the sample space, the collection of all events, and the probability measure respectively, and E be a mathematical expectation symbol.

Assume that the following conditions are satisfied:

- **A1**. **a)** For all $i \in N,\ j \in N_t^i$, observation noise $w_t^{i,j}$ are zero-mean, independent identically distributed (i.i.d.) random values with bounded variances: $\mathrm{E}(w_t^{i,j})^2 \le \sigma_w^2$.

  b) Graphs $\mathscr{G}_{B_t}$, $t = 1, \ldots$ are i.i.d. (independent identically distributed), i.e. the random events of appearance of of "time-varying" edge (j,i) in graph $\mathscr{G}_{B_t}$ are independent and identically distributed for the fixed pair (j,i), $i \in N, j \in N_{\max}^i = \cup_t \bar{N}_t^i$. For all $i \in N,\ j \in N_t^i$ weights $b_t^{i,j}$ in the control protocol are independent random variables with mean values (mathematical expectations): $\mathrm{E} b_t^{i,j} = b_{av}^{i,j}$, and bounded variances: $\mathrm{E}(b_t^{i,j} - b_{av}^{i,j})^2 \le \sigma_b^2$. Let B_{av} be the corresponding adjacency matrix.

  c) For all $i \in N$, $t = 1, \ldots$ random values z_t^i are independent with expectations: $\mathrm{E} z_t^i = \bar{z}$ which do not depend on i, and variances: $\mathrm{E}(z_t^i - \bar{z})^2 \le \sigma_z^2$.

  Additionally, all mentioned in Assumption **A1** independent random variables and vectors are mutually independent.
- **A2**. Graph $\mathscr{G}_{B_{av}}$ has a spanning tree (for the consensuses to be achievable throughout the system Chebotarev and Agaev (2009)).
- **A3.** For step-size γ of control protocol (4) the following conditions are satisfied:

$$0 < \gamma < \frac{1}{indeg_{\max}(B_{av})},\ |\delta(\gamma)| < 1, \tag{8}$$

  where $\delta(\gamma) = 1 - \gamma Re(\lambda_{max}(\mathscr{L}(B_{av}))) - \gamma^2 \lambda_{\max}(\mathscr{L}(B_{av})^T \mathscr{L}(B_{av}))$.

4.2 Averaged Models

Let $x_0^\star$, be the weighted average of the initial states

$$x_0^\star = \frac{\sum_i g_i x_0^i}{\sum_i g_i}$$

where g^T is the left eigenvector of matrix B_{av} Lewis et al. (2014) ($x_0^\star = \frac{1}{n}\sum_{i=1}^n x_0^i$ in the case of balanced topology graph $\mathscr{G}_{B_{av}}$) and $\{x_t^\star\}$ is the trajectory of averaged systems

$$x_{t+1}^\star = x_t^\star + \bar{z} - 1. \tag{9}$$

where $\bar{z}$ is expectation defined by Assumption **A1.c**.

4.3 Theoretical result

Consider vector $X_t^\star \in \mathbb{R}^n$, $t = 0, 1, \ldots$ which consists of $x_t^\star$ at all places.

Theorem 1. **If** Assumptions **A1–A3** hold **then** for averaged squared difference $\nu_t = \mathrm{E}||X_t - X_t^\star||^2$ of trajectories of closed-loop systems (5) and (9) following inequalities are satisfied:

$$\nu_t \le \frac{\gamma^2 H + S}{1 - \delta(\gamma)} + (\delta(\gamma))^t \left(\nu_0 - \frac{\gamma^2 H + S}{1 - \delta(\gamma)} \right), \tag{10}$$

$H = \sigma_w^2 ||B_{av}||^2$, $S = n\sigma_z^2$, i.e. **if** additionally $\nu_0 < \infty$, **then** the asymptotic mean square ε-consensus in (5) is achieved with $\varepsilon = \frac{\gamma^2 H + S}{1 - \delta(\gamma)}$.

Proof. The proof is a particular case of the proof in Amelina et al. (2013).

Theorem 2. **If** Assumptions **A1–A3** hold **then** optimal step-size $\gamma^\star$ of control protocol 3 can be calculated by formula:

$$\gamma^\star = -\frac{S}{H}\Delta + \sqrt{\frac{S^2}{H^2}\Delta^2 + \frac{S}{H}} \tag{11}$$

where $\Delta = \frac{Re(\lambda_{\max}(\mathscr{L}(B_{av})))}{\lambda_{\max}(\mathscr{L}(B_{av})^T \mathscr{L}(B_{av}))}$.

Proof. The proof is similar to proof given in Amelina et al. (2015b).

4.4 Step-Size Adjustment

While step-size meets Assumption **A3** the system will converge to the weighted average value. But for the network in different conditions the value of optimal step-size of control protocol is different. It is important to choose the step-size according to conditions under which the network is operating in order to optimize its productivity. Lets evaluate an efficiency of control protocol operation over period of time T with functional $\mathscr{F}(\gamma) = \int_0^T F(\gamma, X_0, w) \approx \frac{1}{N}\sum_{l=1}^N \bar{F}(\gamma, X_0, w)$, $\bar{F}(\gamma, X_0, w) = \frac{1}{T}\sum_{t=1}^T ||X_t - X_t^\star||^2$, where X_0 is the vector of initial agents' states, X_t is the vector of agents' states at time instant t, $X_t^\star$ is the average value of system load at time instant t, w is the noise. $\mathscr{F}$ characterizes both rate of convergence and level of convergence or average deviation of agents' states from the consensus value. Figures 1 and 2 show graphs of dependence of $\mathscr{F}$ on γ. The form of the graphs, particularly the presence of relatively clear minimum, suggests that the value of step-size γ can be adjusted via SPSA-type algorithm.

We use the following procedure for step-size adjustment. The performance of control protocol is evaluated through time interval $k = 1, 2, \dots$ of length T. The step-size value is adjusted at the lapse of time interval $k = 1, 2, \dots$ or once in T time instants. At odd time intervals k the value F_k^0 of functional $\bar{F}$ is computed: $F_k^0 = \frac{1}{T}\sum_{t=2(k-1)T+1}^{(2k-1)T} ||X_t - X_t^\star||^2$. At the end of the odd time interval step-size value is updated according to formula $\hat{\gamma}_k + \beta\Delta_k$. At even time intervals k the value F_k^+ of functional $\bar{F}$ is computed: $F_k^+ = \frac{1}{T}\sum_{t=(2k-1)T+1}^{2kT} ||X_t - X_t^\star||^2$. At the end of even time interval the value of step-size is updated according to the following formula.

$$\hat{\gamma}_{k+1} = Pr_{\left[0.001, \frac{1}{indeg_{max}B_{av}} - 0.001\right]}\left(\hat{\gamma}_k - \alpha\Delta_k \frac{F_k^+ - F_k^0}{\beta}\right), \quad (12)$$

where α and β are parameters of the algorithm which should be chosen considerably small, $\hat{\gamma}_k$ is the estimate of step-size at k-th time interval, Δ is a random value with Bernoulli distribution which takes values ± 1. Computed value of step-size is projected on interval $\left[0.001, \frac{1}{indeg_{max}B_{av}} - 0.001\right]$ to fulfill constraint (8).

5. SIMULATION RESULTS

Let's consider network on $n = 20$ agents connected as a undirected "double circle" i.e. agents are also connected with second order neighbors in a circle (node i is connected with $i-1$, $i+1$, $i-2$ and $i+2$). Amount of tasks coming to the system at time instant t is a Poisson random variable and distributed with parameter $\sigma_z = 2$. Complexity of task is a random variable with uniform distribution on interval $[8, 12]$. Agent productivities p^i, $i = 1 \cdots n$ are constant and have values distributed uniformly in interval $[0.5, 1.5]$. Noise occurring during information exchange between agents $w_t^{i,j}$ is a random variable with uniform distribution on interval $[-0.1, 0.1]$. Let's say at time instant t_0 all agents have equal queue lengths with 100 tasks.

Let's change step size of control protocol every 100 time instants, i.e. $T = 100$. We take $\gamma_0 = 0.4$, $\alpha = 1.5 \cdot 10^{-5}$, $\beta = 5 \cdot 10^{-3}$. Fig. 1 shows graph of dependence of $\mathscr{F}$ computed after first time interval $k = 1$ on step size γ for system with agents connected as "double circle". The averaging is done on data

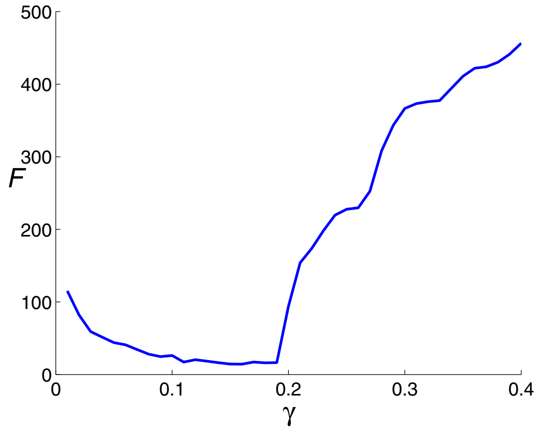

Fig. 1. Dependence F on γ for ring-type topology.

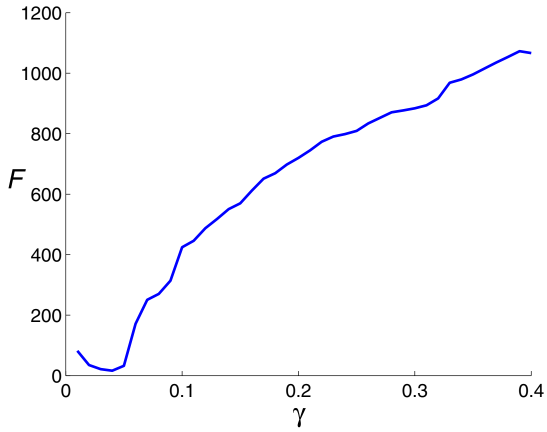

Fig. 2. Dependence F on γ for full-graph topology.

from $N = 10$ experiments with random initial loads X_0 evenly distributed in interval $[50, 100]$.

Compute optimal step size for described system according to formula (11). In the system $\sigma_w^2 = 0.0033$, $||B_{av}||^2 = 80$, $\sigma_z^2 = 1.72$, $Re(\lambda_{max}(\mathscr{L}(B_{av}))) = 6.2361$, $\lambda_{\max}(\mathscr{L}(B_{av})^T\mathscr{L}(B_{av})) = 38.8885$. Substituting given values to formula (11) we get 0.1604.

At time instant $t = 5000$ system topology changes to full graph that changes step size considerably. Agents now have information about states of all nodes in the network and large step size is not needed for optimal system performance. Fig. 2 shows averaged graph of dependence of $\mathscr{F}$ computed after first time interval $k = 1$ on step size γ for system with full graph topology. The averaging is done on data from $N = 10$ experiments with random initial loads X_0 evenly distributed in interval $[50, 100]$.

Compute optimal step size for described system according to formula (11). For new topology $||B_{av}||^2 = 380$, $Re(\lambda_{max}(\mathscr{L}(B_{av}))) = 20$, $\lambda_{\max}(\mathscr{L}(B_{av})^T\mathscr{L}(B_{av})) = 400$. Optimal step size equals 0.05.

Fig. 3 shows the step size of control protocol chosen by stochastic approximation procedure. On figure x-axis stands for number of step size adjustments or number of time instants T. At first step-size converges to optimal value for system with

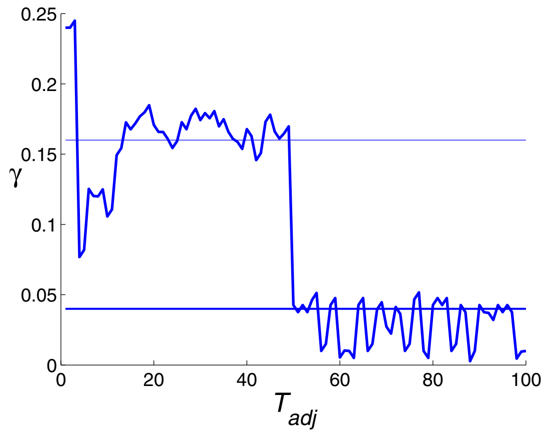

Fig. 3. Step-size adjustment.

“double circle” topology and after topology changes it begins to take values around new optimal value.

6. CONCLUSION

In this paper we showed a way to adjust step-size of local voting protocol using stochastic approximation type algorithm. The network model was assumed to have switched topology, noise in measurements. We introduced a control strategy (a modification of a local voting protocol) for load balancing of network system and algorithm for choosing its step-size. A simulation of the system operating by introduced control strategy in changing conditions is provided. Optimal values of step-sizes obtained analytically are given.

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