

## ABSTRACTS

UDK 517.977, 519.173

*Abramovskaya T. V., Petrov N. N.* **On the monotonicity of the search number in the Golovach problem** // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 3–9.

The Golovach problem also known as the  $\varepsilon$ -search problem is as follows. A team of pursuers and an evader are on the topological graph. The goal of the pursuers is to catch the evader, in other words to approach the evader for a distance not greater than a given nonnegative number  $\varepsilon$ . It is supposed that the evader is invisible to the pursuers and he is well informed about the search plan of the pursuers. The problem is to find the  $\varepsilon$ -search number — the minimum number of the pursuers who can capture the evader. The following property of the  $\varepsilon$ -search number is under study: let  $G$  be some graph,  $H$  be its connected subgraph, then the  $\varepsilon$ -search number of the graph  $G$  is not less than the  $\varepsilon$ -search number of the subgraph  $H$ . We call this property the monotonicity. It is known that the  $\varepsilon$ -search number is monotone for the case of trees for each nonnegative  $\varepsilon$ . The monotonicity of the edge-search number, which is equal to the 0-search number, is also known. The sufficient condition of the monotonicity of the number of any graph is proven. The improvement of this result for the case of complete subgraphs is performed. The Golovach function for graphs obtained by removing one edge from the complete graphs with unit long edges is constructed.

*Keywords:* guaranteed search,  $\varepsilon$ -capture, search numbers, the Golovach function.

*Bibliogr.* 3 references. Fig. 3.

UDK 517.977

*Barabanov A. E., Romaev D. V.* **Limiting optimal adaptive filtering with unknown disturbance covariance** // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 10–18.

Variance of the Kalman—Bucy filter estimation error depends essentially on the correct choice of the covariance matrices of plant disturbance and measurement noise. What is the most difficult in filters design is the lack of necessary statistical information about the useful signal and the disturbance. A filter is called adaptive if the necessary statistical parameters are tuned on-line during the active estimation process. The problem of adaptive filtering under parametric uncertainty conditions is studied in the paper. A method of the limiting optimal Kalman—Bucy filter design is presented for the case of unknown matrix of disturbance covariance. The adaptive algorithm of estimation of the disturbance covariance matrix is formulated and based on stochastic approximation. Conditions of the algorithm convergence are established. Simulation results illustrate the adaptive filter behavior.

*Keywords:* optimal filtering, Kalman—Bucy filter, stochastic approximation.

*Bibliogr.* 7 references. Fig. 3.

UDK 513.6

*Bakulin S. V., Vavilov N. A.* **On the subgroups normalized by  $EO(2l, R)$**  // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 19–27.

We show that description of subgroups in the general linear group  $GL(n, R)$  normalized by a classical group is terribly much harder than previously assumed. For the case of even orthogonal groups we carry out a thorough level calculation. It shows that, even under additional assumption  $2 \in R^*$ , the level of a subgroup  $H \leq GL(2l, R)$ ,  $l \geq 3$ , normalized by  $EO(2l, R)$ , is determined by three ideals  $(A, B, C)$  in  $R$ , rather than by two ideals, as was generally assumed. These ideals satisfy inclusions  $C^2 \leq A = B \cap C$ , such triples are called admissible. Here  $A$  is the level of  $H$  with respect to linear transvections  $t_{ij}(\xi)$ , whereas  $B$  is the level of  $H$  with respect to orthogonal transvections  $T_{ij}(\xi)$ . It is slightly harder to describe the third component. In an appropriate realization the Lie algebra of the even orthogonal group consists of matrices anti-symmetric with respect to the skew diagonal. Then  $C$  is the level of  $H$  with respect to the complementary invariant subspace, i.e. matrices, symmetric with respect to the skew diagonal. With any admissible triple  $(A, B, C)$  we associate the corresponding relative elementary subgroup  $EEO(2l, R, A, B, C)$ . It is normalized by  $EO(2l, R)$  and, moreover, is  $EO(2l, R)$ -perfect.

*Keywords:* general linear group, orthogonal group, subgroups, normalized by a given subgroup, levels, elementary transvections, orthosymplectic transvections.

*Bibliogr.* 23 references.

UDK 519.165+168

*Gracheva P. V. Method of discrete optimization in multidimensional dichotomic data structuring via Grassmannian parametrization // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 28–37.*

The article deals with the combinatorial optimization problem appearing in certain multidimensional dichotomic data clustering approaches. A method to reduce large computational time based on algebraic properties of finite geometries is considered. The vector parametrization of  $Gr_2(k, n)$  Grassmannian is proposed. Such parametrization makes it possible to minimize the amount of memory required for the computation and to reduce the number of operations. A greedy algorithm based on this parametrization and on the Gray codes is proposed. This algorithm allows us to use parallel processing for further reducing the computation time.

*Keywords:* information aggregation, categorial data, Grassmannian parametrization, computation time reduction.

Bibliogr. 11 references.

UDK 519.214

*Korchevsky V. M. On the strong law of large numbers for a sequence of random variables without the independence condition // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 38–41.*

New sufficient conditions for the applicability of the strong law of large numbers are established for a sequence of dependent random variables  $X_1, X_2, \dots$  with finite variances. No certain type of dependence between random variables of the sequence is assumed. Classical condition  $\sum_{n=1}^{\infty} DX_n (\log_2 n)^2 / n^2 < \infty$  is used. This condition is contained in different theorems on the strong law of large numbers for a sequence of random variables without the independence condition.

*Keywords:* strong law of large numbers, sequences of dependent random variables.

Bibliogr. 9 references.

UDK 519.8

*Krivulin N. K. An extremal property of the eigenvalue for irreducible matrices in idempotent algebra and an algebraic solution to a Weber—Rawls location problem // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 42–51.*

An extremal property of the eigenvalue for irreducible matrices in idempotent algebra is investigated and it is shown that the eigenvalue appears to be the minimum value of a functional defined with the matrix on the set of vectors having nonzero elements. The minimax single facility location problem (the Weber—Rawls problem) on the plain with the rectangular metric is considered. An appropriate representation of the problem is given in terms of idempotent algebra and a new algebraic solution is proposed that is based on the investigation results of the extremal property of the eigenvalue and reduces to evaluation of the eigenvalue and eigenvector of a matrix.

*Keywords:* idempotent semifield, vector semimodule, eigenvalue and eigenvector of a matrix, the Weber—Rawls location problem, rectangular metric.

Bibliogr. 15 references.

UDK 519.816

*Kusherbaeva V. T., Sushkov Yu. A., Tamazian G. S. Scales and methods of deriving priorities in the analytic hierarchy process // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 52–60.*

The scales and the priorities deriving the methods used in the analytic hierarchy process (AHP) are theoretically researched. Results of the study are formulated as theorems that are applicable for the decision-maker and allow him to choose proper parameters of the AHP for solving real-world problems.

*Keywords:* decision analysis, analytic hierarchy process, scales, eigenvalue method, geometric mean method.

Bibliogr. 12 references. Tab. 2.

UDK 519.71

*Reitmann F., Slepukhin A. S. On the upper estimates of the Hausdorff dimension for negatively invariant sets of local cocycles // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 61–70.*

We consider negatively invariant sets of local cocycles which are generated by nonautonomous ordinary differential equations. Using the singular values of the linearized flow and adapted Lyapunov functions we give upper Hausdorff dimension estimates for a class of such negatively invariant sets.

*Keywords:* Local cocycle, nonautonomous system, negatively invariant set, Hausdorff dimension.

*Bibliogr.* 12 references.

UDK 519.7

*Selivanov A. A. Dynamics of quantum entropy maximization for finite level systems // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 71–79.*

The problem of statistical dynamics model construction for finite level quantum mechanical system is considered. To do that we apply the Maximum Entropy Principle (MEP) that was formulated by E.T. Jaynes in 1957: *the entropy of any physical system tends to increase until it achieves its maximum value under constraints imposed by other physical laws*. Following to this principle we consider the von Neumann entropy as a goal function and apply speed-gradient principle to derive the dynamical equation that describes evolution of the density operator in finite level systems. In this case physical constraints will be a mass conservation law and an energy conservation law. Stability of the obtained system equilibrium points is investigated. By using LaSalle's theorem it was shown that the density function tends to Gibbs distribution, where the entropy reaches its maximum value. The method is illustrated by an example, where we consider finite amount of identical particles distributed between cells. The results of numeric simulation are presented.

*Keywords:* entropy, dynamics, extremal principles, speed-gradient.

*Bibliogr.* 16 references. Fig. 2.

UDK 519.626

*Tikhomirov A. S. On the convergence rate of an algorithm of the Markov non-homogeneous random search for an extremum // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 80–89.*

This paper is devoted to the theoretical study of the convergence rate of an algorithm of the Markov non-homogeneous random search for an extremum. Random search methods have been successfully used for solving complicated optimization problems. Nevertheless, there are few theoretical results on the convergence rate of these algorithms.

Suppose that the objective function  $f$  takes its minimal value at a single point  $x_*$ . We use a random search to find the global minimizer  $x_*$  with the prescribed accuracy  $\varepsilon$ . We measure the convergence rate of such an algorithm by the number of evaluations of the objective function required to attain the desired accuracy  $\varepsilon$  of the solution. In this paper, we obtain theoretical estimates of the rate of convergence and then use them to construct asymptotically fast optimization methods. It is shown that, if the objective function is «non-degenerate», then the number of its evaluations required to obtain the desired accuracy  $\varepsilon$  in the solution has the order  $O(|\ln \varepsilon| \ln |\ln \varepsilon|)$  as  $\varepsilon \rightarrow 0$ .

Note that many local optimization algorithms (such as steepest descent method) need  $O(|\ln \varepsilon|)$  steps to attain  $\varepsilon$ -neighbourhood of  $x_*$ , but require much more strong restrictions on an objective function. In global optimization problems the order for the number of iterations is typically worse; it is  $O(1/\varepsilon^\alpha)$  for some  $\alpha > 0$ . Thus the constructed Markov random search is asymptotically fast. Its asymptotic rate of convergence is just marginally worse than the rate of convergence of a standard descent algorithm for an ordinary local optimization problem.

*Keywords:* random search, global optimization, stochastic optimization.

*Bibliogr.* 17 references.

UDK 539.30

*Tovstik T. M. On the solution of systems of linear algebraic equations by Gibbs's method // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 90–98.*

An algorithm of approximate solution of a system of linear algebraic equations by the Monte Carlo method in combination with the ideas of Gibbs and Metropolis for fields simulation is presented. A solution in the form of the Neumann's series is estimated. The vector of solutions is found at once. A dimension of the system may be large. The co-variation matrix for the separate imitation is found. The presented method is similar to the method described in the paper by S. M. Ermakov and A. I. Rukavishnikova (2009).

By examples of systems of the third and 100th orders the comparison of exactness of the given method with the method of S. M. Ermakov and A. I. Rukavishnikova and with the classic method of Monte Carlo consisting in the consecutive estimation of the unknown vector components is performed.

*Keywords:* linear algebraic equations, Monte Carlo method, simulation of fields by Gibbs method.  
Bibliogr. 4 references.

UDK 531.011

*Byachkov A. B., Yushkov M. P. A tensor form of the Udwadia—Kalaba equations of motion for nonholonomic systems* // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 99–108.

In their well-known work “A new perspective on constrained motion” F. E. Udwadia and R. E. Kalaba have derived equations of motion for nonholonomic systems that didn’t include constraint reaction forces. The number of these equations is equal to the number of generalized coordinates of a system. Udwadia and Kalaba suppose that the equations obtained by them are the simplest and moreover most comprehensive so far discovered. These equations are derived with the help of the E. Moore inverse, which was proposed in 1920 and generalized in 1955 by R. Penrose. The equations are written in a compact matrix form, but nevertheless it is difficult to use them in practice because of the poorly known Moore—Penrose inverse.

In the paper offered a tensor form of the Udwadia—Kalaba equations of motion for nonholonomic systems is given, which is simple and illustrative. It is derived as a result of substitution of expressions for generalized reaction forces, which are given by the second group of the Maggi equations, into the Lagrange equations of the second kind with multipliers.

*Keywords:* the Lagrange equations, the Udwadia—Kalaba equations, nonholonomic systems.  
Bibliogr. 10 references. Tab. 1.

UDK 531.3

*Dmitriev N. N. The motion of a rigid body along a horizontal plane with a thin annular domain of support and with orthotropic friction* // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 109–117.

The motion of a rigid body along a horizontal plane with a thin annular domain of support is considered. It is assumed that the friction between the body and the plane has orthotropic properties. A theorem on the dependence of the phase trajectories on the moment of inertia of a rigid body and the components of the friction tensor has been formulated and proved.

*Keywords:* anisotropic friction, orthotropic friction.  
Bibliogr. 9 references.

UDK 533.601; 533.5

*Miroshin R. N. Pairwise summation series in the inverse problem of local interaction theory* // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 118–122.

The semiempirical theory of the local interaction is used in continuum mechanics for fast approximations of forces and moments of bodies moving in it. Empirical coefficients in this theory are the same for a large class of bodies and can therefore be determined by known forces and moments of several bodies to be used in calculation of the reaction medium for others (the inverse problem). Often it is not possible to find such coefficients for one body and one has to consider two bodies at last. Solving the inverse problem in the latter case reduces to the pairwise summation series. In this article we show how these series are combined into a single system of linear algebraic equations.

*Keywords:* local interaction theory, inverse problem, pairwise summation series.  
Bibliogr. 9 references.

UDK 533.6.011.72

*Mostovyykh P. S., Uskov V. N. Compatibility conditions on a weak discontinuity in axisymmetric flows of non-viscous gas* // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 123–133.

Supersonic steady axisymmetric flows of a non-viscous non-heatconductive perfect gas are considered. The desired gasdynamic parameters are the Prandtl—Meyer function, the velocity vector inclination angle and the total pressure logarithm. The compatibility conditions on the weak discontinuity are obtained. They have the form of assembled algebraic and ordinary differential equations with respect to differences of the first derivatives of the gasdynamic parameters (the Prandtl—Meyer function, the velocity vector inclination angle and the total pressure logarithm) on the both sides of the discontinuity surface. These equations are solved in a particular case of a weak gasdynamic discontinuity in a uniform initial flow. It is

shown that as the discontinuity approaches the axis of symmetry, the first derivatives of the gasdynamic parameters grow infinitely.

*Keywords:* weak discontinuity, supersonic axisymmetric flows, compatibility conditions.  
Bibliogr. 3 references. Fig. 3.

UDK 534.1:531.36

*Pasynkova I. A., Stepanova P. P. Precessions of an unbalanced Jeffcott rotor in massive compliant supports // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 134–141.*

An influence of dynamical properties of massive compliant supports on whirling motion of an unbalanced Jeffcott rotor has been studied. It has been shown that symmetrical hyperboloidal and conic precessions can exist under some conditions. The dynamic response with two additional non-linear resonances, which are connected with dynamics of massive bearings, have been obtained. The linear standard method of stability investigation has been applied.

*Keywords:* Jeffcott rotor, hyperboloidal and conic precessions.  
Bibliogr. 8 references. Fig. 3.

UDK 524.7-1/-8

*Morozova D. A., Troitskiy I. S. Multiwavelength Study of Gamma-Ray Bright Blazars // Vestnik St.Petersburg University. Ser. 1. 2011. Issue 4. P. 142–149.*

We have investigated total intensity radio images of 6 gamma-ray bright blazars (BL Lac, 3C 279, 3C 273, W Com, PKS 1510-089, and 3C 66A) and their optical and gamma-ray light curves to study connections between gamma-ray and optical brightness variations and changes in the parsec-scale radio structure. We use data obtained with radiointerferometer VLBA, optical ground-based telescopes LX200, AZT-8, Perkins, MAPCAT, SMARTS, and the Cosmic Gamma-ray Observatory Fermi (LAT). The authors have modelled the radio maps, performed optical observations, and constructed the gamma-ray light curves. Over the period from August 2008 to November 2009, superluminal motion is found in all 6 objects with the apparent speed ranging from  $2c$  to  $32c$ . The blazars with faster apparent speeds, 3C 273, 3C 279, PKS 1510-089, and 3C 66A, exhibit stronger variability of the gamma-ray emission with maximum flux density of gamma-ray flares up to  $8,84 \cdot 10^{-6}$  ph/s\*cm<sup>2</sup>. Only blazars with faster apparent speeds show simultaneous optical and gamma-ray brightness variations during the flares. Our results are consisted with models in which the gamma-ray emission arises in high-relativistic jets.

*Keywords:* blazar, gamma-ray, relativistic jet.  
Bibliogr. 13 references. Fig. 5. Tab. 1.

## CONTENTS

### Mathematics

<i>Abramovskaya T. V., Petrov N. N.</i> On the monotonicity of the search number in the Golovach problem.....	3
<i>Barabanov A. E., Romaev D. V.</i> Limiting optimal adaptive filtering with unknown disturbance covariance.....	10
<i>Bakulin S. V., Vavilov N. A.</i> On the subgroups normalized by $EO(2l, R)$ .....	19
<i>Gracheva P. V.</i> Method of discrete optimization in multidimensional dichotomic data structurization via Grassmannian parametrization.....	28
<i>Korchevsky V. M.</i> On the strong law of large numbers for a sequence of random variables without the independence condition.....	38
<i>Krivulin N. K.</i> An extremal property of the eigenvalue for irreducible matrices in idempotent algebra and an algebraic solution to a Weber—Rawls location problem.....	42
<i>Kusherbaeva V. T., Sushkov Yu. A., Tamazian G. S.</i> Scales and methods of deriving priorities in the analytic hierarchy process.....	52
<i>Reitmann F., Slepukhin A. S.</i> On the upper estimates of the Hausdorff dimension for negatively invariant sets of local cocycles.....	61
<i>Selivanov A. A.</i> Dynamics of quantum entropy maximization for finite level systems.....	71
<i>Tikhomirov A. S.</i> On the convergence rate of an algorithm of the Markov non-homogeneous random search for an extremum.....	80
<i>Tovstik T. M.</i> On the solution of systems of linear algebraic equations by Gibbs's method.....	90

### Mechanics

<i>Byachkov A. B., Yushkov M. P.</i> A tensor form of the Udvardia—Kalaba equations of motion for nonholonomic systems.....	99
<i>Dmitriev N. N.</i> The motion of a rigid body along a horizontal plane with a thin annular domain of support and with orthotropic friction.....	109
<i>Miroshin R. N.</i> Pairwise summation series in the inverse problem of local interaction theory.....	118
<i>Mostovyykh P. S., Uskov V. N.</i> Compatibility conditions on a weak discontinuity in axisymmetric flows of non-viscous gas.....	123
<i>Pasynkova I. A., Stepanova P. P.</i> Precessions of an unbalanced Jeffcott rotor in massive compliant supports.....	134

### Astronomy

<i>Morozova D. A., Troitskiy I. S.</i> Multiwavelength Study of Gamma-Ray Bright Blazars.....	142
-----------------------------------------------------------------------------------------------	-----

<b>Abstracts</b> .....	156
------------------------	-----

<b>List of the articles</b> .....	161
-----------------------------------	-----