

ABSTRACTS

UDK 517.956:517.983.246

Durante T., Cardone G., Nazarov S. A. Modeling junctions of plates and rods by means of self-adjoint extensions // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 3–14.

Based on the asymptotic analysis of elliptic problems in thin domains and their junctions, a model is constructed for the mixed boundary value problem with a scalar second-order differential equation in the union of thin plate and rods. One of the ends of each rod is jointed to the plate and the other is supplied with the Dirichlet conditions, but the Neumann conditions are imposed on the remaining part of the junction boundary. The asymptotic expansion of the solution of this problem possesses a few distinguishing features, that is, coefficients in the expansion are rational functions in the large parameter $|\ln h|$ ($h \in (0, 1]$ implies the small geometrical parameter) and the solution of the limit problem on the longitudinal cross-section of the plate gets logarithmic singularities at the points where the rods are adjoined to the plate. That is why the standard formulations of boundary value problems do not fit to describe the asymptotics while the techniques of self-adjoint extensions and function spaces with detached asymptotics are employed.

It is established that an adequate model of the boundary value problem in the three-dimensional junction serves as a problem on the hybrid domain, namely the union of a two-dimensional domain and one-dimensional segments. This problem relies on an abstract equation with the operator implying a special self-adjoint extension of the matrix operator composed from the operators of the limit problems for the plate and the rods. Parameters of the self-adjoint extension are determined by examining the boundary layer effect in junction zones.

Estimates are obtained for the accuracy of the approximation of the solution to the original singular perturbed problem and the solution to the problem in the hybrid domain.

Keywords: junction of thin bodies, scalar mixed boundary value problem, hybrid domain, self-adjoint extension.

Bibliogr. 37 references. Fig. 3.

UDK 519.245+519.683

Ermakov S. M. Construction of parallel algorithms in problems of applied mathematics // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 15–22.

Using an example of solving linear algebraic systems of equations, this article shows that algorithms with important properties of parallelism and asynchronism can be constructed after reducing an initial problem to a problem of computation of continual integral (integral over trajectories). Previous works of the author and his colleges mention that parallel and asynchronous algorithms of the Monte Carlo method can excel the corresponding iterative algorithms with a growth of system's dimension n . This article presents a qualitative explanation of this fact.

A solution of a linear algebraic system of equations $X = AX + F$ permits a simple representation as an integral over trajectories only under condition $\lambda_1(A) < 1$, where $\lambda_1(A)$ is the maximal modulo eigenvalue of matrix A . If this condition is not fulfilled then one can construct recurrent algorithms for solving the initial system, which possess properties of (coarse-grained) parallelism. However this case requires additional conditions of synchronization of the algorithm.

Finally, basing on results achieved by the author and W. Wagner, the article describes a way to construct effective analogues of stochastic algorithms — algorithms of the quasi Monte Carlo method which excel the Monte Carlo method in speed of convergence.

The similar approach can be applied to a wide class of mathematical and theoretical physics problems, where integral representations of solutions are known.

Keywords: parallelism, Linear algebraic systems of equations, Monte Carlo methods, Quasi-Monte Carlo methods.

Bibliogr. 15 references.

UDK 517.938

Zuber I. E., Gelig A. Kh. Stability of uncertain systems // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 23–30.

A continuous system

$$dx/dt = A(\cdot)x$$

is considered where the elements $A(\cdot)$ of $m \times m$ -matrix are bounded and are functionals of arbitrary form. The variation bounds of coefficients are known only. It is assumed that the local theorem on solution existence is valid and for all $t > 0$ any solution, which remains in bounded domain, is continuable.

Constructing a Lyapunov function in terms of quadratic form with the Jacobian matrix of coefficients, the relations between the variation bounds of system coefficients, for which the system is globally exponentially stable, are obtained.

The pulse system is also considered, which is obtained from the original one by means of replacing elements on the main diagonal by synchronous pulse modulators realizing the amplitude-frequency modulation. When the pulse frequency tends to infinity after averaging in time the signals on modulators outputs this system reduces to the system considered above.

For the pulse system, conditions for variation bounds of matrix coefficients and a lower bound of pulse frequency such that the global asymptotical stability of the system is provided, are obtained.

Keywords: uncertain systems, stability, Lyapunov function, puls systems.

Bibliogr. 9 references.

UDK 518:517.432.1

Kabardov M. M. Geometric interpretation of the Euler–Knopp summation method in the Laplace transform inversion problem // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 31–36.

A method of inversion of the Laplace transform $F(s) = \int_0^\infty e^{-st} f(t) dt$ which consists in representation of the original function by the Laguerre series

$$f(t) = \sum_{k=0}^{\infty} a_k L_k(bt). \quad (1)$$

is considered. Some conformal mapping of the plain (s), depending on a parameter ξ , the value of which is in accordance with the geometry of the image function singularities locations is preliminarily made. So (1) takes the form

$$f(t) = \frac{b-\xi}{b} \exp(\xi t) \sum_{k=0}^{\infty} c_k L_k((b-\xi)t).$$

It's shown, that such a technique is equivalent to an application of the Picone–Tricomi method with subsequent acceleration of the convergence of series (1), based on the nonlinear Euler–Knopp procedure

$$\sum_{k=0}^{\infty} a_k z^k = \sum_{k=0}^{\infty} A_k(p) \frac{z^k}{(1-pz)^{k+1}}, \quad A_k(p) = \sum_{j=0}^k \binom{k}{j} (-p)^{k-j} a_j.$$

In this technique the original is represented by the series

$$f(t) = \exp\left(\frac{bpt}{p-1}\right) \sum_{k=0}^{\infty} \frac{A_k(p)}{(1-p)^{k+1}} L_k\left(\frac{bt}{1-p}\right),$$

the parameters ξ and p being related by an equality $p = \xi/(\xi - b)$.

In contrast to other summation methods there is no need in investigation of regularity conditions in the scheme suggested.

Keywords: inversion of the Laplace transform, Laguerre series, acceleration of the convergence, Euler—Knopp method.

Bibliogr. 6 references. Fig. 2.

UDK 519.63

Krivulin N. K. Evaluation of the Lyapunov exponent for generalized linear systems with the exponential distribution of elements of a transition matrix // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 37–47.

A second-order generalized linear stochastic dynamical system is considered. The evolution of the system is described through a dynamical equation with a random transition matrix, which is linear in idempotent algebra with the operations of taking maximum and addition. It is assumed that some entries of the transition matrix are equal to zero, whereas the other entries are independent and exponentially distributed. The problem of evaluation of the Lyapunov exponent defined as the mean asymptotic growth rate of the system state vector is considered. The known results for the problem are restricted to the system with a matrix which has zero entries on its diagonal. In order to evaluate the Lyapunov exponent for the case of matrices which can have a zero row, zero diagonal entries, or only one zero entry, an approach is applied based on construction and analysis of a sequence of one-dimensional distribution functions. The Lyapunov exponent is evaluated as the mean value of the random variable determined by the limit distribution of the sequence.

Keywords: stochastic dynamical system, random matrix, idempotent algebra, Lyapunov exponent, convergence of distribution functions.

Bibliogr. 9 references.

UDK 517.972.2:517.974.8

Krym V. R. The Euler—Lagrange method as formulated by Pontryagin // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 48–58.

The classical variational problem with the nonholonomic restrictions can be solved by the Euler—Lagrange method as formulated by Pontryagin, but the Lagrange multipliers will be measurable functions only. We propose the modified Euler—Lagrange method where the Lagrangian of the initial problem involves the independent components of the velocity vector only. Then the Lagrange multipliers prove to be an absolutely continuous vector-function. This method is applied to the problem of horizontal geodesics for a nonholonomic distribution on a manifold. We established that these equations contain two types of connection, connection on a distribution and connection on a manifold, and this fact was not taken into account by other authors.

Keywords: Euler—Lagrange method, Pontryagin maximum principle, classical variational calculus, horizontal geodesics, nonholonomic distribution.

Bibliogr. 8 references.

UDK 518

Lebedinskaya N. A., Lebedinski D. M. Triangulation refinement by the edge split operation // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 59–62.

The fact that every triangulation of a plane polygonal area can be refined by multiple use of the edge split operation in such a way that 1) the maximal diameter of the triangles is less

than any a priori fixed positive number and 2) the minimal internal angle of the triangles of the refinement of the triangulation is greater than the minimal internal angle of the triangles of the original triangulation, divided by 9. The refinement in question is built in two stages. First, the triangulation is refined in such a way that the triangles of the refinement obtained can be paired, only boundary triangles can be without a pair; at this stage every triangle can be subdivided at most by 4 parts. Then, the triangulation is refined once more to obtain a refinement with the maximum diameter of the triangles less than the fixed ε . At every stage, the minimal internal angle of the triangles is decreased not more than by a coefficient of 3. This is guaranteed by the lemma that states the internal angles of triangles in which a triangle is subdivided by a median, are greater than one third of the minimal angle of the original triangle.

Keywords: triangulation refinement, local transformations, angles of the triangles.

Bibliogr. 2 references.

UDK 517.968.2+517.956

Maz'ya V. G., Poborchi S. V. Unique solvability of the integral equation for a harmonic potential of a simple layer on the boundary of the domain with a peak // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 63–73.

It is known that validity to write the solution to the Dirichlet problem for the Laplace equation in the form of a simple layer potential $V\varrho$ with unknown density ϱ leads to the equation $V\varrho = f$ for finding ϱ , where f is the Dirichlet boundary data. We consider an n -dimensional ($n > 2$) bounded domain with the vertex of an isolated peak on its boundary S and show that the operator V^{-1} , defined on smooth functions on S , can be uniquely extended to an isomorphism between the trace space on S of functions with the finite Dirichlet integral over \mathbf{R}^n and its dual space. Thus, the equation $V\varrho = f$ is uniquely solvable with respect to ϱ for any trace $f = u|_S$ of a function u having the finite Dirichlet integral over \mathbf{R}^n . By using an explicit description of the above trace space, one can state a theorem on solvability of the boundary integral equation $V\varrho = f$ in terms of the function describing the peak near its vertex.

Keywords: Dirichlet problem, Laplace equation, boundary integral equation, harmonic potential, domain with a nonsmooth boundary.

Bibliogr. 12 references.

UDK 519.21

Miroshin R. N. On the solution of the Chapman—Kolmogorov integral equation represented as a series // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 74–78.

The nonlinear integral equation, mentioned in the title, is the basic one of the theory of random Markov processes. Solution thereof is the density of transition probability function. Usually this equation is solved by means of reducing it to a linear equation. In 1932 S. N. Bernstein tackled the problem of solving it directly. In 1962 O. V. Sarmanov found such solutions for the stationary Markov process in terms of bilinear series. In 2007 the author obtained some solutions represented as integrals of the product of two kernels of known integral transformations. Free from Sarmanov restrictions, in this paper there are derived the solutions as a series, the terms of which contain the product of two orthogonal functions. The results are illustrated with examples, in which the series converges to a simple function.

Keywords: Chapman—Kolmogorov integral equation, random Markov processes, probability density.

Bibliogr. 7 references.

UDK 518:517

Stoyanova S. B. The invariant cubature formula of ninth degree for a hypercube // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 79–85.

The Sobolev theorem is used to construct cubature formulae for integrals of the hypercube in \mathbf{R}^n :

$$C_n = \{x = (x_1, x_2, \dots, x_n) \in \mathbf{R}^n \mid -1 \leq x_i \leq 1, i = 1, 2, \dots, n\},$$

which are exact for all polynomials of degree not exceeding 9 and invariant with respect to the group of all orthogonal transformations of the hyperoctahedron

$$G_n = \left\{ x = (x_1, x_2, \dots, x_n) \in \mathbf{R}^n \mid \sum_{i=1}^n |x_i| \leq 1 \right\}$$

onto itself.

Section 1 is an introduction to the subject and contains a review of known results. In sections 2 and 3 we derive the parameters of the cubature formulae for $n \geq 4$ and $n = 3$, respectively. Numerical results (the nodes and the coefficients of cubature formulae) are presented in section 4.

Keywords: cubature formulae, degree of accuracy, group of transformation, hypercube, hyperoctahedron.

Bibliogr. 7 references. Tab. 1.

UDK 517.586

Kholshevnikov K. V., Shaidulin V. Sh. Asymptotics of the uniform norm of associated Legendre functions P_n^k (the case $k \ll n$) // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 86–93.

The uniform (Chebyshevian) norm of the associated Legendre harmonics $P_n^k(x)$ on the orthogonality segment $-1 \leq x \leq 1$ is investigated. The main goal is to examine an asymptotic behaviour while the subscript tends to infinity. The case of a fixed superscript, or one increasing slower than $n^{2/3}$, is considered. It is obtained that the norm increases as n^k . A corresponding coefficient is equal to the greatest value of the Bessel function of order k . The asymptotics of the coefficient is also established: decreasing as $k^{-1/3}$ with an exact numerical coefficient. Direct calculations show that the asymptotic approximation is satisfactory even for the first small values of the superscript.

Keywords: associated Legendre functions, uniform and mean-squared norm, asymptotics.

Bibliogr. 14 references. Fig. 1. Tab. 1.

UDK 539.3

Lebedev A. V. Buckling of plates weakened with holes // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 94–99.

This article is devoted to analysis of buckling of a thin isotropic elastic rectangular plate, weakened with a central square hole, under axial compressive loading. The effect of the plate and cut-out geometric parameters and boundary conditions on the value of the critical load and buckling mode is studied.

Three types of boundary conditions are considered. For cylindrical bending the critical buckling load decreases when the area of the hole increases. For the plate with simply supported edges the critical buckling load decreases for the buckling modes with odd wave numbers in longitudinal direction and increases for the buckling modes with even wave numbers. For the plate with clamped edges the critical buckling load increases as the area of the hole grows regardless of the buckling mode.

For rather long plates the critical buckling load doesn't depend on the hole area if the hole is not too large.

Keywords: buckling, plate with the cut-out.

Bibliogr. 4 references. Fig. 6.

UDK 533.601.18

Memnonov V. P., Morozov A. O. An experimental estimation of statistical characteristics for the surface roughness // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 100–104.

With the help of the atomic force microscope Solver PRO-M by using an additional averaging over 25 points for the noise reduction, measurements of the rough surfaces for two samples of mica and silicium were carried out with the accuracy on the height up to 0.02 nm. Through the histograms of the angle inclinations of the intercepts along the measurement line under the minimal horizontal displacements of 10 nm the distribution density $f(\beta)$ of the deflection angles β for the normals of the micro areas from the normal to the mean level of the surfaces of these samples was estimated. It turned out that for the greater part of the angles β range the function $f(\beta)$ is approximately represented by the exponent $\exp(-m * |\beta|)$. And merely on the fewer part for smaller angles $\beta < \beta_b$ the square dependence of the exponent $f(\beta) \sim \exp(-c * \beta^2)$ appears. The experimental procedure and the error estimation of the constants m and c for mica and silicium are presented. It should be noted that the thick adsorption layer was present on the surfaces of all the samples. It filled also all surface cavities as the measurements were made under the atmospheric pressure in order to reproduce the conditions of air flows in modern high technological devices, which have in some directions small nanometre dimensions as it would be realized, for instance, in new winchesters now under design. Therefore the obtained results are not applicable to clean surfaces, for which the experiments with high vacuum are necessary.

Keywords: statistical characteristics of surface roughness, atomic force microscope, distribution of the deflection angles for normals.

Bibliogr. 6 references. Fig. 3.

UDK 539.3:519.63

Morozov N. F., Tovstik P. E. The rod dynamics under longitudinal impact // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 105–111.

As a rule in problems of the rod stability under dynamical loading it is supposed that the compressing force is constant with respect to the longitudinal coordinate. In the paper the short longitudinal impact on the rod end is studied. It is supposed that the impact time is shorter than the time during which the longitudinal wave goes two times the rod length. The nonlinear boundary problem is reduced approximately to two linear problems. In the first of them the wave equation is solved and the axial force depending on the axial coordinate and on the time is found. In the second problem for the given axial force by using the Fourier expansion the lateral rod vibrations connected with the initial rod imperfections are constructed. It is established at first that the lateral vibrations essentially depend on the amplitude and on the mode of the initial imperfections. Secondly the lateral vibrations amplitude unboundedly grows when impact is finished. It is clear that this approximate solution is acceptable only for the small enough time interval. The more exact nonlinear system of equations is obtained. This system allows us to estimate the range of applicability of the linear approximate approach described here.

Keywords: rod dynamics, longitudinal impact.

Bibliogr. 6 references. Fig. 2.

UDK 539.3, 517.928

Filippov S. B. Buckling of an annular plate subjected to a radial tension along the inner circle // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 112–121.

Buckling of an annular plate under action of the radial tensile load uniformly distributed on the inner circle of the plate is considered. Dependence of critical loading on boundary conditions is investigated. The analytical solution of the equation describing buckling of an annular plate has been obtained in the work by Mansfield. However this work has essential defect. Initial stresses used in it do not satisfy the boundary conditions which usually occur in practice.

In this work the equation of buckling of an annular plate for the real initial stresses equal to zero on the outer circle of the plate is obtained. This equation has no analytical solution. In general case boundary value problems for definition of critical loadings and buckling forms are solved by the method of shooting. For narrow and wide plates by means of asymptotic methods the approximate formulas for calculation of critical loading are obtained. It is shown, that for a narrow plate the replacement of initial stresses of Mansfield with real stresses can lead to reduction of critical loading in 20 times.

The problem of buckling of an annular plate is of interest in connection with buckling of a stiffened shell of rotation because such plate can be considered as a model of the supporting frame of the shell.

Keywords: annular plate, buckling, boundary value problem, asymptotic method.

Bibliogr. 4 references. Fig. 3.

UDK 517.947, 534.1, 536.241

Yufereva L. M., Lavrov Yu. A. The thermo-mechanical field in multi-layered plate with internal sources // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 122–128.

The approximate solution of one-dimensional problem on determination of coupled temperature oscillations and mechanical vibrations in multi-layered plate is built by the small-parameter method. The processes in the plate are excited by internal heat sources that are distributed within its volume. In the planes dividing each pair of adjacent layers the conditions of ideal heat and mechanical contact are fulfilled. Mechanical and thermal regimes on external plate surfaces submit to the Winkler's and Newton's models respectively. There are derived the approximate expressions for eigenvalues of the Sturm–Liouville problem that are necessary in building the basis functions for thermal and vibration field's decomposition.

Keywords: multy-layered plate, ideal heat and mechanical contact, approximate analytic solution.

Bibliogr. 4 references. Fig. 1.

UDK 520.87, 519.25

Baluev R. V. On the detection of periodic components in observational data // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 2. P. 129–136.

The paper deals with the methods of periodic signal detection, basing on the periodograms constructed using the least-squares principle. These methods include, for instance, such widely used tools like the Lomb–Scargle periodogram and the multi-harmonic periodogram. The major attention is paid to the recent important author's results obtained in the problem of estimation of the statistical significance of candidate periodicities, which are found using these periodograms. The approximations obtained are quite useful in practice and simultaneously are rather accurate.

Bibliogr. 14 references.