

UDK 517.938

Zuber I. E., Gelig A. Kh. **Stability of uncertain discrete-time systems** // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 3–9.

The uncertain system

$$x_{n+1} = A_n x_n, \quad n = 0, 1, 2, \dots,$$

is considered.

The coefficients $a_{ij}(n)$ of $m \times m$ -matrix A_n are arbitrary functionals and satisfy the following conditions

$$\begin{aligned} |a_{i,i}(n)| &\leq \alpha_* < 1, \\ |a_{i,j}(n)| &\leq \alpha_0, \quad j \geq i + 1, \\ |a_{i,j}(n)| &\leq \delta, \quad j < i. \end{aligned}$$

In particular, one of such systems is a switching-type system where the matrices A_n can take any values from a given finite set.

Constructing a special Lyapunov function, the estimate $\delta \leq \delta(\alpha_0, \alpha_*)$ is obtained, which guarantees global asymptotical stability. In particular, a system is stable if the latter inequality is replaced by $a_{i,j}(n) = 0$ for $j < i$.

It is shown that pulse-width modulation (PWM) systems can be reduced to the uncertain systems considered above. In the cases of PWM of type 1 and type 2 the coefficients of A are functions of $x(n)$ and functionals, respectively.

Keywords: discrete-time systems, global asymptotical stability, pulse-width modulation.

Bibliogr. 12 references.

UDK 514.822:514.762.5:514.752.8+530.12:531.51:537.1

Krym V. R., Petrov N. N. **The principal bundles and the problem of topological quantization of charges** // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 10–17.

In this paper the space of admissible particle velocities is assumed to be the four-dimensional nonholonomic distribution on the principal or associated bundle. Equations of the horizontal geodesics for this distribution are the same as the equations of motion of a charged particle in the general relativity theory. It is proved that if we «add» a 4-torus to the elementary particle physics Standard Model Lie group, then the wavefunctions will be eigenfunctions of the operators of charges and the horizontal lift will not depend on the coupling constants. The wavefunctions constructed satisfy the well-known Dirac equation and its generalizations. For such wavefunctions the topological quantization of the electric, leptonic and baryonic charges is fulfilled.

Keywords: nonholonomic distributions, linear connection, curvature tensor, Einstein equations, Kaluza–Klein theory, topological quantization.

Bibliogr. 24 references. Fig. 1. Table 1.

UDK 518

Lebedinskaya N. A., Lebedinski D. M. **Multiresolution analysis for Zlamal approximation** // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 18–22.

For the Zlamal approximation (piecewise continuous approximation of degree less or equal to two) the new space of approximating functions is proved to contain the old one for a refinement of the triangulation. The multiresolution expansion formulas, which express the old basis functions as linear combination of the new ones, are explicitly written for the case of adding a vertex to one of the edges. We consider both the cases of adding a vertex to a boundary edge or an internal one. The formulas obtained can be used as well if we add multiple vertices to triangles sufficiently far from

each other, since the triangulation transformation in question and its influence on the coefficients of the expansion of approximating function with respect to the standard Zlamal basis, are local. Local bases of a complimentary summands W in the decomposition of the new approximating function space into the direct sum of the old one and W , also for the cases of adding a vertex to a boundary edge or an internal one. For the bases indicated the decomposition and reconstruction formulas of the wavelet transformation are explicitly written. All the formulas obtained were verified by the computer algebra system MuPAD 2.5.3 for Linux.

Keywords: Zlamal approximation, multiresolution analysis, decomposition formula, reconstruction formula.

Bibliogr. 4 references.

UDK 519.6, 533.6.011

Memnonov V. P. Testing of random number generators via simulation of a problem with exact solution // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 23–32.

Through numerical Monte Carlo simulation of a problem with known exact solution the linear multiplicative generators with the modulus $M = 2^{31} - 1$ were tested for the use in parallel calculations. Obtained final temperatures departures from known theoretical values in the simulated problem were compared with possible Monte Carlo errors produced by finite samples used. In addition with the help of the sample correlation coefficients it was estimated the true correlation coefficients between the rotational temperatures, obtained on different processors and with different multipliers on each of them. Also in the case when on processors the additional samples in the final state were calculated in order to enlarge general sample. For this simulated random temperature variables by partly averaging were transformed into approximate normal variables. Then for them the true correlation coefficients were estimated from the sample correlation coefficients. It was found that generators with 204 different Fishman-Moore's multipliers in parallel work show best Monte Carlo quality, with all the departures being below values of theoretical Monte Carlo errors. No correlations between random variables produced by generators with different multipliers were revealed in this case. So that apparently generators with different multipliers produce independent sequences of pseudorandom numbers. Yet if some additional samples on every processor were taken, as it often used in order to enlarge the general sample, the correlations inside them appeared with the values of Monte Carlo errors being overshooted by temperature departures for multipliers from the low part of Fishman—Moore's rating in many such cases. Therefore they should not be used in similar manner.

Keywords: random number generators, parallel codes, Monte Carlo method, correlation coefficients.

Bibliogr. 16 references. Fig. 4.

UDK 517.93+681.511.42

Smirnova V. B., Utina N. V., Shepelyavyi A. I., Perkin A. A. Frequency—domain estimates for the number of slipped cycles in a phase system with vector nonlinearity // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 33–43.

Two classes of phase control systems with vector nonlinear functions are considered. They are systems described by ordinary differential equations and systems described by difference equations. Their main feature is the presence of vector periodic nonlinear function in mathematical description of the system. The problem of cycle slipping is investigated. For the both classes of control systems the frequency—domain estimates for deviation of each angular coordinate from its initial value are obtained. The direct Lyapunov method with periodic Lyapunov functions is exploited. By means of the Yakubovich—Kalman lemma all the results are formulated in terms of the transfer function of the linear part of the system. The results have the form of frequency—domain inequalities with varied parameters which satisfy algebraic inequalities.

Keywords: phase systems, Lyapunov method, Yakubovich—Kalman lemma, cycle slipping.

Bibliogr. 16 references.

Tovstik T. M. **On the criteria when estimating the cubature formulas residue** // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 44–53.

The cubature formulas for calculations of integrals of functions $f(X)$, $X = (x_1, \dots, x_n)$ which are given in the n -dimensional unique hypercube $K^n = [0, 1]^n$ and have the integrable mixed derivatives of the type $\partial_{x_1^{\alpha_1}, \dots, x_n^{\alpha_n}} f(X)$, $0 \leq \alpha_j \leq 2$ are studied. Residues of cubature formulas $R[f] = \int_{K^n} f(X) dX - \sum_{k=1}^N c_k f(X(k))$, $c_k > 0$ are estimated depending on weights c_k and assemblies $X(k)$, and on properties of functions $f(X)$. The residue is estimated through the integrals of derivatives of functions $f(X)$ in the r -dimensional ($r \leq n$) boundaries of hypercube K^n in the form $|R[f]| \leq \sum_{\alpha_j} G(\alpha_j) \int_{K^r} |\partial_{x_1^{\alpha_1}, \dots, x_n^{\alpha_n}} f(X)| dX_r$, where coefficients $G(\alpha_j)$ are the criteria, depending on the parameters c_k and $X(k)$ only. The algorithm of these criteria calculation in the two-dimensional and in the n -dimensional cases is given. Some examples are studied. The discrepancy is the partial case of the criteria, and the algorithm presented is the generalization of algorithms which are used for the discrepancy calculations. The results of the paper may be used for the cubature formulas optimization depending on the parameters c_k and $X(k)$.

Keywords: cubature formulas, residue estimate, discrepancy.

Bibliogr. references.

UDK 669.018:539.5

Arutyunyan R. A. **The embrittlement problem in material science** // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 54–58.

In material science two lines of experimental investigations of embrittlement of metallic materials are outlined. The works of the first line deal with the estimation of «brittleness» in dynamic loadings through the definition of absorbed energy when the specimen with a crack is fractured and the experiments in which the fracture toughness, the resistance of material to fast crack propagation, is studied. The disadvantages of these methods are widely discussed in scientific literature. The second line is rather traditional, when the material embrittlement is studied in compact specimens. In this case the value of relative deformation and the value of relative changes of cross section area of a specimen at a fracture are used as the embrittlement characteristics. As follows from our considerations, the traditional definition of these parameters accepted in the strength of materials textbooks and in numerous publications is not quite correct, so they can't describe the real degree of material embrittlement. To improve the situation we introduce a new definition of embrittlement parameters based on the mechanical and physical considerations.

Keywords: embrittlement of metallic materials, relative deformation, relative changes of cross section area.

Bibliogr. 15 references.

UDK 533.6.011

Babarykin K. V., Kouzmina V. E., Matveev S. K., Petrova V. N. **Research of self-oscillations in the impacting jets** // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 59–67.

A mechanism of self-oscillations in the impacting jets is investigated. The numerical simulation of the self-oscillations process in the underexpanded supersonic jet flowing on the plant limited obstacle is carried out. The creation of an adequate phenomena physical model is carried out by a transition from common to quotient method. Namely, the self-oscillations in the impacting jets is considered as a partial case of a vast range of problems about self-oscillations, arising when a non-homogeneous supersonic stream flowing an obstacle with separated zone beginning. The new working out phenomena physical model taking into account a hi-enthalpy gas flowing from periphery flow into the separated zone and the interaction of processes in the periphery flow and separated zone. The self-oscillations mechanism is connected with periodic flowing of hi-force gas from periphery flow into the separated zone and following gas outflow from it. The alternation of flow into and outflow phases is conditioned by the changing of separated zone dimensions. The

increase of separated zone dimensions leads to a cessation gas flow into and beginning of gas outflow from the separated zone. The decrease of separated zone dimensions in the outflow phases following to what the periphery flow again falls on the obstacle blocking the gas outflow from the separated zone. Further process is repeated. Interaction of processes in the separated zone and in the periphery flow is showed mainly by a presence of rear waves associating to shock waves in the adjacent flow regions.

Keywords: stream, flow, self-oscillations, obstacle, rear waves to shock waves.

Bibliogr. 13 references. Fig. 5.

UDK 533.601.1

Bogatko V. I., Kolton G. A., Potekhina E. A. About the peculiarity of Lagrange variables application in the problem of the hypersonic flow past the body // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 68–75.

The characteristic peculiarities of application of Lagrange variables in the problem of the hypersonic flow past the body is considered. It is marked that it's appropriately in the intensive shock waves problems to select the particle parameters values not at the surface $t = t_0$ ($t_0 = \text{const}$), but at the surface $t = \sigma$, where σ is that time moment, when the particle intersects the discontinuity surface. By the example of the solution of the two-dimensional flow problems of the moving with a high variable velocity plane and axisymmetric bodies it is shown how the transition to Lagrange variables allows to get the system of equations which describes the gas flow past the strong shock wave and suits for the thin shock layer method. The solution is under construction in the form of the series by the powers small parameter describing the relation of the gas densities on the head shock wave front. It is marked that all nonlinear effects of problem are in the equation for the definition of the gas particle motion law in the zeroth-order approximation. It is show the cases for which this equation may be integrated. The solution is wrote in the quadratures for the other unknown quantities. The rearrangement process of the gas flow in the shock layer is investigated. The zone, in which the flow reconstruction occurs, is marked out. This zone existence time definition condition (the time of new flow regime establishment) was received. In the special case of the transfer from the wedge uniform motion to the uniformly accelerated motion the establishment time of the uniformly accelerated motion is defined from the quadratic equation.

Keywords: gas dynamics, flow past the body, hypersonic flow, Lagrange variables, thin shock layer.

Bibliogr. 7 references.

UDK 539.374

Lashkov V. A., Matveev S. K. Changing of roughness of a surface under impingement of solid particles // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 76–82.

The surface of a body under impingement of solid particles of two-phase flow is subject to deformation and destruction and, thus, changes its roughness. The depth of a crater, which originates from the impact of a solid particle, is determined on the base of semi-empirical theory of penetration of a spherical pellet into the surface of a target. The inertial penetration of an absolutely rigid sphere into a metallic half-space of the target is considered. The expression for maximum depth of the penetration of a spherical striker into a half-space in a wide range of dimensionless parameters changing is obtained. The obtained data of calculation of penetration depth and experimental data are compared.

The evolution of a surface roughness during the action of solid particles of two-phase flow is reviewed. The value of probability of a particle fall into a crater on the surface left by previous particles is obtained. The estimation is conducted on how fast the surface of the model is covered by the craters under impingement of solid particles. It is revealed, that some seconds of presence of the experimental model in two-phase flow are enough for the solid particles to begin to drop on the surface that has been deformed by the previous particles. Therefore, when defining experimentally

the coefficients of restitutions under interaction of solid particles of two-phase flow with the model it is necessary to allow for a surface roughness changing, which originates from impacts of particles.

Keywords: roughness of a surface, penetration depth, probability of a particle fall into a crater.

Bibliogr. 14 references. Fig. 4.

UDK 531.383

Lestev M. A., Tikhonov A. A. Nonlinear effects in dynamics of micromechanical gyros // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 83–88.

Microelectromechanical vibrating gyros of LL and RR types are considered. Nonlinear dependence between the forces of elasticity in supports and the displacements of sensitive elements is taken into consideration as well as nonlinear dependence between the electrostatic forces and the displacements of sensitive elements. The nonlinear differential equations of a gyro, functioning in the measuring mode, are evaluated. These equations contain both analytical and non-analytical nonlinearities. The influence of revealed nonlinearities on the dynamics and precision of vibrating gyros is investigated. The stable steady-state vibrations of a gyro are obtained with the use of the averaging technique. The corresponding resonant curves are constructed. The obtained results can be used in developing the devices of the types considered.

Keywords: micromechanical gyros, microelectromechanical gyros, dynamics, sensitive elements.

Bibliogr. 8 references. Fig. 2.

UDK 539.3, 517.928

Malysheva O. M., Filippov S. B. Influence of eccentricity on buckling of thin ring-stiffened cylindrical shells under action of external pressure // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 89–98.

Last years numerical methods of calculation of thin-walled structures developed very intensively. However their use in problems of dynamics and stability of the stiffened shells is connected with certain difficulties. Therefore asymptotic methods till now keep leading positions in the theory of the stiffened shells.

Problem of definition of optimal parameters of ring-stiffened cylindrical shells, providing the maximal value of critical pressure of the structure with the fixed weight, have been solved by asymptotic methods for case when the centre of gravity of the ring cross-section section is on a middle surface of the shell. However usually the distance between the centre of gravity of the cross-section and the middle surface, named the eccentricity, is distinct from zero because the rings are located inside or outside of the shell.

In this paper by means of asymptotic methods the approximate formulas for the critical external pressure for ring-stiffened cylindrical shells in case of the distinct from zero eccentricity are obtained. It is shown, that formulas of Bryant and Kendrick recommended in the USA for engineering calculations, give overestimated values of the critical pressure. The algorithm for estimation of an optimal arrangement of rings and sizes of their rectangular cross-sections is developed. Values of optimum parameters for some thin-walled structures are found.

Keywords: buckling, stiffened shell, optimal parameters, asymptotic method.

Bibliogr. 7 references. Fig. 4.

UDK 531.1:629.76

Novoselov V. S. Optimal trajectories of the tangential contact // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 99–103.

The variation method of the optimization of coplanar trajectories of the tangential transfer with the prescribed relative velocity and taking into account the duration of the active sections is proposed. An analytical construction of three successive approximations in the problem of transfers between coplanar orbits with small eccentricities is given. The expressions are presented only up to

the third order. The analytic expressions for the minimum characteristic velocity requirements and the optimal direction of the thrust are discussed.

Keywords: the optimization of coplanar trajectories, the problem of transfers between coplanar orbits, optimal direction of the thrust.

Bibliogr. 9 references.

UDK 532.5.032

Pavlovsky V. A., Nikushchenko D. V. The rheological model for computation of flows in a wide range Reynolds numbers // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 104–112.

In the article a new approach to create a phenomenological model of fluid motion is given, which is alternative to the mixing length hypothesis by L. Prandtl. This approach allows us to describe fluid motion independently of the flow regime in the area of fluid. On a base of the approach we created a new differential model of fluid motion, which can be used for the both flow regimes — «Uniform laminar-turbulent model» (ULTM). So the field of scalar parameter of turbulence is introduced, which is the ratio of turbulent shear stress to full stress value in a case of simple shear flow. This makes it possible to write a new turbulent viscosity equation. The influence of scalar parameter of turbulence on a flow is taken into account with the help of the additional transport equation. The model is suitable for compressible and incompressible fluid, and allows us to obtain analytical solutions in quadratures for simple shear flows. Different forms of the system of equations of motion with boundary conditions are given.

Keywords: Reynolds equation, fluid motion, turbulent model, turbulence, turbulent viscosity.

Bibliogr. 16 references.

UDK 539.3

Pronina Yu. G. Equal mechanochemical corrosion of an ideal elasto-plastic hollow sphere under constant pressure // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 113–122.

This paper is concerned with the equal-rate surface mechanochemical corrosion of a thick-wall sphere subjected to constant internal and external pressure of aggressive environment. Sphere material is assumed to be ideal elasto-plastic with taken into account changes of mechanical characteristics. Corrosion rates are considered to be inversely as the exponent of time (when closed oxide layer appears and leads to corrosion rate reduction). Furthermore, according to the most of experimental investigations corrosion rates linearly depend on stress intensity. In the article the differential equation for the stress intensity increasing in elastic deformation stage has been derived. The solution of this equation has been found. Formulas for the sphere wall thickness at any time have been expressed. An analytical solution of the problem when corrosion rate does not depend on stress has been achieved. Yield conditions of an ideal elasto-plastic sphere under pressure have been formulated. An algorithm to determine theoretical lifetime of the vessel subjected to pressure of aggressive surrounding has been developed. Estimating functions for durability prediction have been proposed. Several examples have been examined in the paper. Some remarks on the article [5] have been given.

Keywords: aggressive environment, mechanochemical corrosion, elasto-plastic deformation, hollow sphere, lifetime, durability prediction.

Bibliogr. 7 references.

UDK 533.6.011

Ryabinin A. N., Irissou E., Legoux J.-G., Moreau C. Heat exchange of a supersonic jet with an obstacle in the cold spray // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 123–128.

Time-resolved thermographic measurements are performed by a high speed infrared camera on a substrate surface during the cold gas dynamic spraying without powder particles. Experiments

were carried on the commercial cold spray system spanning a wide range of gas temperatures and nozzle transverse speeds. The substrate surface temperatures were measured directly beneath the cold spray nozzle as well as at predetermined distances from the nozzle, giving a general evaluation of the heat input to the substrate in actual deposition conditions. The mathematical model of the heat exchange is proposed. Some model parameters are determined from the experiments. Computer program is created for heat exchange calculation. The results of calculations are in accordance with the experiments in various gas stagnation temperatures and various nozzle transverse speeds.

Keywords: time-resolved thermographic measurements, mathematical model of the heat exchange.

Bibliogr. 3 references. Fig. 4.

UDK 532.70; 542.12; 532.5:532.135

Tsibarov V. A. The stochastic method in the vascular hemodynamics // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 129–138.

The modified stochastic model of blood is propounded. The mass and volume distributions of the blood particles are taken into account. The coalescence — disintegration processes of the particles and magnetic field influence are taken into account too. A new general complete and closed system of hemodynamic equations is propounded. It is based on the stochastic equation. The variants of solving the RSE-problem and capillary vascular problem are suggested. The boundary problem in hemodynamics is formulated. The parameters of particle — particle interaction potential are calculated. The volume distribution, the poly-particles collisions and attractive potential influence on the viscosity coefficients and self-diffusion coefficient are obtained. The quality properties of the blood and its structural elements are adduced. The polytropic equation and formula for sound velocity of polytropic media are obtained. It is shown that the number density of aggregating blood is described by a logistic curve. The algorithm of deriving the Burnett corrections on the stress tensor and self-diffusion vector from its expressions for dense gas or gas-solids suspensions is presented.

Keywords: stochastic (kinetic) method, distribution function, fluid-particles systems, fluid-solids suspension, hemodynamics, transport equations, blood vessel, vascular, polytropic, polytropic curve, non-newtonian medium, coalescence, aggregation, coagulation, structural elements of the blood, transport coefficients, rheology.

Bibliogr. 20 references.

UDK 521.1:531.011

Kuznetsov E. D., Kholshchikov K. V. The orbital evolution of the two-planet system the Sun — Jupiter — Saturn // Vestnik St.Petersburg University. Ser. 1. 2009. Issue 1. P. 139–150.

The orbital evolution of the two-planet system the Sun — Jupiter — Saturn is examined. The Hamiltonian function is represented in terms of osculating elements referred to Jacobian coordinates, and then developed in the Poisson series in all osculating elements. Averaging of the Hamiltonian with respect to the mean longitudes of Jupiter and Saturn by the Hori—Deprit method is fulfilled using echeloned Poisson series preserving terms of the third order with respect to the small parameter (Jupiter to the Sun mass ratio). The generating function of the averaging transform and the relations between osculating and mean elements, which are induced by it, are found up to the second degree of the small parameter. Numerical integration of averaged equations is carried out on the time span of 10^{10} years. The planetary motion is almost periodic. The difference between the first and the second approximations has an order of the square root of the small parameter which points out the presence of a weak resonance. It is shown that taking into account the third order terms weakly influences a qualitative picture of motion, but it is necessary for the due determination of the main periods.

Keywords: Planetary system, orbital evolution, averaging, Hori—Deprit method.

Bibliogr. 17 references. Fig. 3. Table 3.