

ABSTRACTS

UDK 517.935.4

Blinov E. N., Sokolov B. M. **Stability, estimation of state and control in a certain nonlinear system** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 7–14.

The controlled system of nonlinear ordinary differential equations is considered. In this system the non-stationary linear part, the part linearly dependent on the control, and the nonlinearity vanishing on the stationary solution and satisfying the Lipschitz condition are allocated. Besides, there are restrictions on the control. In the beginning the simple model is considered, and the problem of stabilization with use of a stationary condition is solved. It is supposed that all parameters of the object are known (such as the value of a state at the initial moment of time, coefficients of right-hand sides of the system equations) and its state is measured. The control is calculated from the condition of obtaining the given stationary value of a state. For this purpose the initial value of the control is projected on the crossing of some linear subspace and the convex area of restrictions on the control. This subspace is obtained by equating the right-hand side of the system to zero. The theorem of stabilization basing on the I. G. Malkin theorem of stability for permanent disturbances is proved. Then it is supposed, that some parameters are unknown, and the state of object is not measured either. For simultaneous estimation of unknown parameters and the state of the object the method of recurrent target inequalities developed by V. A. Yakubovich is used. The infinite system of linear inequalities is solved component-wise in the final number of steps with respect to estimations of unknown parameters. The theorem of convergence of the method of estimation and attainment of the target of control is proved.

Bibliogr. 9 references.

UDK 517.929

Gelig A. Kh. **Stability of nonlinear discrete-time systems in the simplest critical case** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 15–18.

Nonlinear time-invariant discrete-time systems of arbitrary orders are considered. Right-hand sides of the equations are sums of polynomials of arbitrary degree and continuous functions, which are infinitely small with respect to the highest-degree terms of polynomials. It is assumed that the characteristic polynomial of the linearized system has one root equal to unity, while its other roots lie inside the unit disk.

By construction of some special polynomial Lyapunov functions, the conditions for instability and asymptotic stability of equilibrium are obtained. To prove asymptotic stability a positive definite Lyapunov function, the increment of which is negative definite along the system's solutions is used. To establish the instability conditions an indefinite Lyapunov function with a negative definite increment is considered.

Key words: nonlinear discrete-time systems, critical case, asymptotic stability, instability.

Bibliogr. 8 references.

UDK 517.929

Zuber I. E. **The extension of class of stabilizable discrete-time nonlinear systems** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 19–24.

The extension of class of stabilizable discrete-time nonlinear systems is performed in three steps. At the first step a nonlinear discrete-time system such that its elements are bounded above, the elements of the first off-diagonal are separated from zero, and the elements above the first off-diagonal are zeros, is considered. The control is supposed to be a scalar state feed-back. At the first

step the vector of control distribution is the last unit vector. The Lyapunov function is constructed as a positive-definite quadratic form with a constant diagonal matrix. The feed-back vector that provides the global stability for the closed-loop system is found.

At the second step it is supposed that the elements of the system matrix above the first off-diagonal are bounded functions. The upper bounds of these elements, for which the stabilizing control of system at the first step is the stabilizing control of system at the second step, are found.

At the third step the extension of the considered vector of control distribution is performed. The norm of a new vector of control distribution, for which the system with the matrix regarded at the second step is asymptotically globally stable, is estimated.

Key words: discrete-time systems, feed-back stabilization.

Bibliogr. 7 references.

UDK 517.925.51

Leonov G. A., Kuznetsov N. V., Kudryashova E. V. Limit cycles of two-dimensional systems. Calculations, proofs, experiments // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 25–61.

One of the central problems in studying small cycles in the neighborhood of equilibrium is a computation of the Lyapunov quantities.

While the first and second Lyapunov quantities were computed in general form in the 40–50s of last century, the third Lyapunov quantity was computed only in the certain special cases.

In the present work the general formulas for computation of the third Lyapunov quantity are given. Together with the classical Lyapunov method for computation of Lyapunov quantities, which is based on the passage to the polar coordinates, the method, developed for Euclidian coordinates and in the time domain, is suggested.

The computation of Lyapunov quantities by two different analytic methods with the use of the modern software tools of symbolic computing permits us to show that the formulas obtained for the third Lyapunov quantity are correct.

The computation for quadratic systems such that the first and second Lyapunov quantities are equal to zero and the third one is not equal zero were carried out. In these computations the quadratic system was reduced to the Lienard equation and by the latter the domain of parameters, corresponding the existence of four cycles: three «small» and one «large» cycles, was evaluated. This domain extends the domain of parameters, obtained for the quadratic system with four limit cycles due to Shi in 1980.

Bibliogr. 31 references. Fig. 7.

UDK 517,9:539:692.445

Tovstik P. E., Shekhovtsov A. S., Shekhovtsov V. A. On the Mises girder // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 62–67.

The Mises girder gives the simplest example of the elastic system, which demonstrates the non-linear behavior, the bifurcation, and the non-unique equilibrium state. The classical Mises girder consists of two elastic compressible rods attached at a slight angle to each other. The typical mode of its loss in stability under the vertical forces is the buckling into the mirror-reflected form. The critical force is equal to the limiting point on the curve «force—deflection».

In this paper by using the Bernoulli—Euler model the bending of linearly elastic rods is taken into consideration in addition. As a result the bifurcation of equilibrium of the straight form of the compressed rods is possible. The interaction of two possible buckling modes is investigated, namely the mode with limiting point on the curve «force—deflection» and the mode of bifurcation of the rectilinear rod form. The critical force is found for the various values of the girder parameters, forces distribution, and boundary conditions. The approximate analytical solution is compared with the exact numerical solution which describes the non-linear bending of rods.

Bibliogr. 8 references. Fig. 3. Table 1.

UDK 514.822:514.752.8

Krym V. R., Petrov N. N. **The curvature tensor and the Einstein equations for the four-dimensional nonholonomic distribution** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 68–80.

We assume that the space of possible particle velocities is the four-dimensional nonholonomic distribution on a manifold of higher dimension, for example, $M^4 \times \mathbb{R}^1$. This distribution is defined by the 4-potential of the electromagnetic field. The equations of admissible (horizontal) geodesics for this distribution are the same as the equations of motion of a charged particle in the general relativity theory. This distribution has a metric tensor with the Lorentzian signature $(+, -, -, -)$, therefore, the causal structure appears as in the general relativity theory. We introduce the covariant differentiation (linear connection) and the curvature tensor for the distribution. The Einstein equations are obtained from the variational principle for the scalar curvature of the distribution. We have proved that the Dirac operator for the 4-dimensional distribution can be expanded on the functions defined on the manifold $M^4 \times S^1$, where S^1 is a circle. For such functions the topological quantization of the electric charge is fulfilled.

Bibliogr. 37 references.

UDK 519.6

Makarov A. A. **Normalized trigonometrical splines of Lagrange's type** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 81–87.

B_φ -splines, which are non-polynomial generalization of the well known B -splines, are investigated. The source of B_φ -splines is the approximation relations regarded as a system of linear algebraical equations, from which polynomial and non-polynomial splines are derived. Normalized trigonometrical splines of Lagrange's type (of zero height) of third order defined by generating vector-function $\varphi(t) = (1, \sin t, \cos t, \sin 2t)^T$ are constructed. The splines obtained are twice continuously differentiable and have the minimal compact support. The system of biorthogonal functionals to B_φ -splines is constructed. The solution of interpolation problem generated by the mentioned biorthogonal system is derived in the space of B_φ -splines.

Bibliogr. 12 references. Fig. 1.

UDK 539.43

Arutyunyan A. R., Zimin B. A., Sud'enkov Yu. V. **The investigation of cyclic durability of construction materials by the method of optical-acoustic spectroscopy** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 88–97.

It is known that under the action of cyclic stresses the effect of fatigue is observed even for the case of low stresses, which can be attributed to gradual damage accumulation. To investigate the fatigue properties the optical-acoustic methods have been used. These methods are based on the definition of elastic characteristics of material, which are connected with the evolution of material structure. Applying these methods to cyclic loadings we can get more information about structure evolution of materials, which defines its cyclic workability.

The investigation of changes in mechanical properties of construction materials during cyclic loading bending tests is presented. Evolution of specimen's properties has been controlled at every 50000 cycles by the method of optical-acoustic spectroscopy. The experiments show that dependences of sound velocity, attenuation, residual flexure and variance of spectral distribution of transmission function on the number of loading cycles is of non-monotone nature. In this case two distinct regions of acoustics parameters changes are observed. Therefore, the durability of materials during cyclic loadings is defined by two main regions—the region of adaptation to the external loadings and the region of damage accumulation. It is shown that using the optical-acoustic spectroscopy method allows us to keep track of integral characteristics of material properties with a

high accuracy. The analysis of frequency dependences of attenuation makes it possible to estimate the character of changes of scale levels of materials structure evolution during cyclic tests.

Bibliogr. 15 references. Fig. 9. Table 1.

UDK 531.011

Byachkov A. B., Zegzhda S. A., Cattani C., Yushkov M. P. **The refined model of acceleration of a car as a problem with a non-retaining constraint** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 97–105.

Systems of differential equations describing acceleration of a car in both cases: with and without slipping of driving wheels are derived. Front-drive and rear-drive cars are considered. The conditions of arising and termination of slipping are discussed. The moment of rolling friction acting on the driving wheels, and their moments of inertia are taken into consideration to obtain the refined dynamical condition of realization of the constraint. In this case we consider a reduced moment of inertia, which includes rotating masses of cardan shaft, details of transmission gear-box, flywheel, crankshaft, etc. connected to the driving wheels, as the moment of inertia of driving wheels. It is noted that this quantity depends, in general, on the relation between angular velocities of driving wheels and engine. The jumps of car acceleration and driving force, which are an interesting peculiarity of car acceleration with slipping of driving wheels, are theoretically found.

Computational results are presented. The calculations obtained by the refined and the simplified dynamical conditions of realization of the constraint imposed on the driving wheels are compared. It is shown that in the latter case the jumps of car acceleration and driving force are observed not only when slipping terminates but also when it starts.

Bibliogr. 3 references. Fig. 6.

UDK 517.958, 539.3

Grekov M. A., Kostyrko S. A. **Generation of a relief on the surface of a film coating under the surface and volume diffusion** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 106–113.

The mechanism of generation of a relief on the surface of a solid with film coating under intensive external heating the surface layer is investigated. The longitudinal remote stresses, surface stress, volume and surface diffusion, and the thickness of the surface layer the elastic properties of which differ from the same properties of the substrate are taken into account. A semi-infinite body with a surface layer is considered under the plane deformation. Wave formation on the surface of the layer is taken up as instability of a flat shape of the surface under the small sinusoidal perturbation of this surface. According to Asaro and Tiller, it is assumed that the chemical potential gradient driving the mass transport along the surface comes from a stress variation along the surface and the surface curvature. Furthermore, it is supposed that volume diffusion that arose under the high temperature due to capillarity and stress variations in the bulk influences the evolution of the surface profile. The flux of diffusing mass through the bulk is determined by the chemical potential as well. Basing on the perturbation technique, the elastic energy at the film surface is derived from the solution of the corresponding problem. The explicit dependence of an undulation amplitude upon the time, physical and geometrical parameters of the problem is obtained. The analysis of influence of this parameters on the critical wavelength perturbed is given.

Bibliogr. 19 references. Fig. 4.

UDK 539.3

Mal'kov V. M., Mal'kova Yu. V. **The plane problem of non-linear elasticity for harmonic material** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 114–126.

The general relations of non-linear plane problem of elasticity are obtained for John's model harmonic material. This model allows us to use to the complete theory of complex potentials applied

to solving the plane problems of linear elasticity. For the specified model of material the analytical solutions are found for the following non-linear singular boundary value problems: Flamant's problem for a half-plane under the point force on the border, the problem of bi-material plane with jumps in stresses and strains on the interface, and the problem of interface crack.

In the nonlinear Flamant problem the nominal stresses do not depend on the parameters of material and have the same root singularity as stresses in the linear Flamant problem. The stresses of other kinds, in particular the true Cauchy stresses, depend on the moduli of elasticity.

As well as the stresses of the linear interface crack problem, the nominal stresses of the nonlinear problem for an interface crack have the root singularity and oscillation at the tips of the crack. The phenomenon of the solution oscillation at the tips of the crack is apparently connected with the use of the harmonic material model, this phenomenon can be absent for some other models, for example, oscillation is absent for incompressible neo-Hookean material. It is known, that displacements in the linear crack problem have the logarithmic singularity at the tips of the crack, which contradicts physics. This singularity remains also in the nonlinear problem for John's harmonic material.

Bibliogr. 24 references.

UDK 519.67

Grigoriev M. I. The construction of a sphere by means of the projective Bezier patches // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 127–131.

The projective (rational) rectangular Bezier patches of second order are determined by nine points in the three-dimensional space and by positive weights ascribed to these points. It is possible to control the form of the patch by varying positions of the control points and values of the weights.

The paper shows the way of sphere construction by means of the projective rectangular Bezier patch of second order. The special set of the control points, three of which are multiple, and suitably chosen weights are used.

Bibliogr. 5 references. Fig. 4.

UDK 531.36:534.1

Evdokimov S. M. Absolute stability of two-dimensional systems with a hysteresis function of relay type // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 132–135.

The autocontrol system

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\alpha y - \beta x - \varphi(t, \sigma), \end{cases}$$

is considered, where $\sigma = ay + bx$, $\alpha > 0$, $\beta > 0$, and $b^2 - \alpha ab + a^2\beta \neq 0$.

$\varphi(t, \sigma)$ is the relay–hysteresis function containing two branches of unambiguous functions $\varphi_1(t, \sigma)$ and $\varphi_2(t, \sigma)$:

$$\varphi(t, \sigma) = \begin{cases} \varphi_1(t, \sigma), & \text{if } \sigma \geq -\delta, \\ \varphi_2(t, \sigma), & \text{if } \sigma \leq \delta \quad (\delta > 0), \end{cases} \quad (1)$$

$$S \leq \varphi_1(t, \sigma) \leq M, \quad -M \leq \varphi_2(t, \sigma) \leq -S \quad (S > 0, M > 0).$$

The phase surface of system (1) with nonlinearity (2) consists of two layers: $P_1 = \{(x, y) : \sigma \geq -\delta\}$ and $P_2 = \{(x, y) : \sigma \leq \delta\}$, which cover each other in the «indeterminacy area» $-\delta \leq \sigma \leq \delta$. One supposes that functions $\varphi_i(t, \sigma) : \mathbb{R}_+ \times \mathbb{R} \rightarrow \mathbb{R}$ ($i = 1, 2$) satisfy the conditions of existence and uniqueness of solution at every point of phase surface.

The definition of absolute stability for two-dimensional systems (1) with relay–hysteresis nonlinearity (2) is formulated, and necessary and sufficient conditions of absolute stability are found using the comparison systems method.

Bibliogr. 8 references. Fig. 1.

UDK 517.9, 531.33, 535.4, 539.3, 550.3

Kiselev A. P., Tagirdzhanov A. M. The Love waves with a transverse structure // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 136–139.

For an arbitrary layered isotropic structure new solutions of elastodynamics problem for surface waves propagation are presented. These solutions describe waves with rectilinear wave fronts propagating with phase velocities of the common *SH*-polarized Love waves. They linearly depend on the lateral transverse variable and have, in addition to the standard *SH* polarization, an anomalous component that is longitudinally polarized. The construction employs an assumption of existence of the standard Love wave. It is based on a potential representation of the wavefield and is quite elementary.

Bibliogr. 6 references.

UDK 539.3

Krakovskaya E. V. On the stress-strain state of a corneoscleral eye shell // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 140–143.

The article contains the comparative analysis of the analytical and numeral solutions for the stress-strain state of a corneoscleral shell under the intraocular pressure.

The corneoscleral eye shell is modeled as two conjugated isotropic or transverse-isotropic uniform spherical shells, the sclera and the cornea, of two different radii.

The effect of the intraocular pressure and shell parameters on the stress-strain state of the corneoscleral shell is studied.

For the first model the sclera and the cornea are assumed to be isotropic and uniform. The analytical solution is obtained by means of the asymptotic integration method by V.Novozhilov when satisfying the continuity conditions on the joint shell line.

For the second model the deformation of the sclera and cornea under intraocular pressure is considered as the deformation of transverse-isotropic conjugated shells. The numerical solution for both models is obtained with the FEM package ANSYS.

The comparison of the results for isotropic and transverse-isotropic conjugated shells shows that anisotropy essentially effects both the amplitude and shape of deformations in the cornea.

The deflection of the cornea considered as the transverse-isotropic spherical shell is larger and its shape of deformations better approximates the actual deformations in the cornea.

Bibliogr. 6 references. Fig. 1. Table 3.

UDK 519.21

Sagatelyan V. K. On a new model of record values // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 3. P. 144–147.

A new mathematical model of record values named «records with confirmation» is constructed. The conception of *k*-th record values, which represents a generalization of ordinary records and order statistics, is a foundation of this new record model. The records with confirmation are considered for arbitrary fixed $k = 1, 2, \dots$, under the assumption that the original random variables X_1, X_2, \dots are independent and have a common continuous distribution function. Two representations are obtained for new record values in the most important particular situations, when *X*'s have exponential or uniform distribution. The expressions for means and variances are found for the uniform and exponential records with confirmation.

Bibliogr. 4 references.