

ABSTRACTS

UDC 519

Demjanovich Yu. K. Minimal Splines and Wavelets // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 8–22.

The paper is dedicated to memory of prominent mathematician S. G. Miklin. Here the Miklin's idea of approximation relations is used for construction of wavelet resolution in the case of spline spaces of zero height. The mentioned approximation relations give the opportunity to establish the embedding of the spline spaces corresponding to enclosed grids. Systems of functionals which are biorthogonal to basic splines are constructed with the relations, and then the obtained systems are used for structure of wavelet decompositions. It is established that for fixed pare grids where one of them is enclosed in other, and for arbitrary fixed (on the coarse grid) spline space there are continual quantity of spline spaces (on the fine grid) which contain the mentioned spline space on the coarse grid. The wavelet decomposition of such inclosure is given and corresponding formulas of decomposition and formulas of reconstruction are deduced. The space of (\mathcal{A}, φ) -splines is introduced with three objects: the full chain of vectors, prescribed infinite grid on real axis and the preassigned vector-function φ with $m + 1$ components (m is called the order of the splines). Under certain suppositions the splines belong to the class C^{m-1} . The caliber relations between basic splines on the coarse grid and basic splines on the fine grid are deduced. General method for construction of biorthogonal system of functionals (to basic spline system) is given. In this way the chain of embedded spline spaces is obtained, and the wavelet decomposition of the chain is discussed. The full classifications of the spaces and of its chains are given in the terms of manifolds.

The manifold of the discussed spaces is identified with the manifold of complete sequences of points in the direct product of interval on real axis and projective space \mathbb{P}^m ; besides a manifold of embedded spaces is identified with the manifold of embedded sequences of the points of the direct product mentioned above.

Bibliogr. 24 references.

UDC 517.5

Dodonov N. Yu., Zhuk V. V. Approximation of functions of two variables in the spaces $L_p(\mathbf{R}^2)$ and $L_p(\mathbf{R}_+^2)$ on directions // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 23–32.

Let $r \in \mathbf{N}$, $\alpha, t \in \mathbf{R}$, $x = (x_1, x_2) \in \mathbf{R}^2$, $f : \mathbf{R}^2 \rightarrow \mathbf{C}$. Let's put

$$\Delta_{t,\alpha}^r(f, x) = \sum_{k=0}^r (-1)^{r-k} C_r^k f(x_1 + kt \cos \alpha, x_2 + kt \sin \alpha).$$

This paper is concerned with the problem, how the behavior of value

$$\left\| \int_E \Delta_{t,\alpha}^r(f, \cdot) \psi_n(t) dt \right\|_{p,G}$$

at $n \rightarrow \infty$, where $E \subset \mathbf{R}$, $G \in \{\mathbf{R}^2, \mathbf{R}_+^2\}$, $\psi_n \in L_1(E)$ — a positive kernel, the norm undertakes in space $L_p(G)$ at $1 \leq p < \infty$, in space $C(G)$ at $p = \infty$ is connected with structural properties of function f . We suppose that these structural properties are characterized by modules of a continuity on directions

$$\omega_{r,\alpha}(f, h)_{p,G} = \sup_{0 \leq t \leq h} \|\Delta_{t,\alpha}^r(f)\|_{p,G}.$$

Here is one of the results which had been received.

Theorem 1. Let E and A be intervals in \mathbf{R}_+ , such that $A \subset E$, $f \in L_p(G)$, $\alpha \in [0, 2\pi]$, when $G = \mathbf{R}^2$, and $\alpha \in [0, \frac{\pi}{2}]$, when $G = \mathbf{R}_+^2$. Let's put $\Delta_{n,k} = \int t^k \psi_n(t) dt$. Then, if at some $r \in \mathbf{N}$ and all $m \in \mathbf{N}$ next relations $\Delta_{m,r} > 0$, $\Delta_{m,r+1} < \infty$ take place and next equalities

$$\lim_{n \rightarrow \infty} \frac{\Delta_{n,r+1}}{\Delta_{n,r}} = 0, \lim_{n \rightarrow \infty} \Delta_{n,r}^{-1} \int_{E \setminus A} \psi_n = 0$$

satisfied then relations

$$\lim_{n \rightarrow \infty} \Delta_{n,r}^{-1} \left\| \int_E \Delta_{t,\alpha}^r(f, \cdot) \psi_n(t) dt \right\|_{p,G} \leq K, \sup_{t \in (0, \infty)} t^{-r} \omega_{r,\alpha}(f, t)_{p,G} \leq K$$

are equivalented.

Concrete methods of approximation are considered. In particular, it is established

Corollary 1. Let p, G, α, f same as in theorem 1,

$$\sigma_{n,\alpha}(f, x) = \frac{2}{\pi n} \int_{\mathbf{R}_+} \Delta_{t,\alpha}^1(f, x) \left(\frac{\sin \frac{nt}{2}}{t} \right)^2 dt.$$

Then relations $\lim_{n \rightarrow \infty} \frac{\pi n}{\ln n} \|\sigma_{n,\alpha}(f)\|_{p,G} \leq K, \sup_{t \in (0, \infty)} t^{-1} \omega_{1,\alpha}(f, t)_{p,G} \leq K$ are equivalent.

Bibliogr. 5 references.

UDC 517.9 + 532.59

Kozlov B. A., Kuznetsov N. G. **A qualitative theory of nonlinear stationary waves on the water** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 33–46.

In the paper the nonlinear boundary problem describing two-dimensional stationary waves on the surface of water with finite depth is discussed. The usual formulation of the problem is applied (the gravity force takes into account, but surface tension is omitted). The last one gives opportunity to discuss the whole of class of bounded waves that include periodic waves, isolated waves and others (for instance, almost-periodic although existence of the last one hasn't established). Mentioned circumstance makes possible to assign the obtained results to the qualitative theory of differential equations (investigation of the properties of solutions without receiving the last ones). In this paper two ways of approach to the qualitative theory are discussed, and the overview is done. The first way is used the average of the solution along vertical sections of the region, and the second one is based on the authors modification of Byatt–Smith's (integro-differential) equation. Thus the paper is given the overview of results obtained for the problem of nonlinear stationary waves on the water with finite depth. Two ways developed for their demonstrations are foundations for the qualitative theory of such waves because of the restrictions of the wave shape are absent (besides of restrictions for boundedness of their profiles and steepness of the last ones).

Bibliogr. 25 references.

UDC 517.95

Koshelev A. I. **On application of universal iterative process to some problems of mechanics** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 47–55.

The iterative universal process introduced by the author some years ago is applied to quasilinear boundary-value problems in elasticity and filtration. The convergency of the method both in weak (energy) and strong ($C^\gamma(\gamma > 0)$) spaces are proved. Some results concerning the existence of weak

and regular solutions are proved. The proof are based on such general results like the Korn inequality for weighted spaces, method of elastic solutions. The main results contain also the Hoelder continuity of displacements for elasto-plastic media with hardening.

Bibliogr. 7 references.

UDC 517.518.238

Kokilashvili V. M., Samko S. G. Singular operators and Fourier multipliers in weighted Lebesgue spaces with variable index // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 56–68.

The paper is devoted to development of Miklin's ideas and results; discussed themes are related to the theory of spaces $L_p^{p(\cdot)}$ with nonstandard growth. Mentioned spaces are called Lebesgue spaces with variable index; they are useful in mechanics, in theory of differential equations, in variational problems. In the paper the boundedness of Fourier multipliers and singular operators in the spaces $L_p^{p(\cdot)}$ are discussed. All theorems are deduced with extrapolation theorem belonging to J. L. Rubio de Francia. Theorems of boundedness of the operators in the Lebesgue spaces with constant indexes are essential in the investigation. Here there are also the theorems for boundedness of maximal Hardy–Littlewood function in the discussed spaces.

Bibliogr. 38 references.

UDC 517.5

Maz'ya V. G., Poborchii S. V. On solvability of the Neumann problem for a planar domain with peak // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 69–85.

We consider the Neumann problem for elliptic quasi-linear equations of second order in a planar domain with the vertex of an outward or inner peak at the boundary. Under certain conditions, the question of solvability of the Neumann problem is reduced to description of the dual space of the boundary trace space $TW_p^1(\Omega)$ for functions in Sobolev class $W_p^1(\Omega)$, $1 < p < \infty$. The dual space mentioned above is characterized in terms of Sobolev classes on Lipschitz curves with negative smoothness exponents and in terms of some function spaces on the interval $(0, 1)$ of real axis. The proof of the principle results of the paper is essentially based on an explicit description of the space $TW_p^1(\Omega)$ for a planar domain with peak which has been found in one of the previous papers by the authors. In particular, necessary and sufficient conditions for q such that the Neumann problem is solvable if boundary function is in $L_q(\partial\Omega)$ are given.

Bibliogr. 8 references.

UDC 519.63

Narbut M. A. Numerical realization of iterative processes applied to the investigation of nonlinear boundary-value problems in the theory of elasticity and filtration // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 86–92.

To solve nonlinear elastic problems for the hardening media we apply first the universal iteration process proposed by A. I. Koshelev in his works devoted to the problem of regularity of solutions for quasilinear elliptic and parabolic systems. In this case we must obtain numerically the solution of the linear elliptic system for all steps of the iterative procedure. The numerical realization is fulfilled in the MATLAB environment using its PDE Toolbox. The finite element procedure was modified in order to obtain the solution of linear algebraic system for every iteration step. Some simple examples were constructed to check the method of computer simulation.

We use also the elastic solutions method for the same nonlinear problems. To deal with this case we replace the Laplace operator in the universal iteration process by Lamé operator of the linear elasticity. It is known that this iteration process converges to the weak solution of the nonlinear problem if the displacements are fixed on the boundary. We consider also some examples with stresses given on the boundary to check the method in this case.

The third part of our paper is devoted to the nonlinear filtration problem. The general properties of the iteration process for the nonlinear parabolic systems were investigated previously by A. I. Koshelev and V. M. Chistjakov. The numerical realization is based on PDE Toolbox procedures which were slightly modified to suit our case. The program was tested for some simple examples.

Bibliogr. 8 references.

UDC 517.586

Kholshevnikov K. V., Shaidulin V. Sh. **The relations between norms of a function and its gradient in classes of surface and solid spherical harmonics in a finite-dimensional space** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 93–96.

The paper contains relations between the Euclidean (mean-square) and the Chebyshev's (uniform) norms of a surface spherical harmonic of order n and its gradient in a tangent bundle of the sphere \mathbb{S}^{k-1} ; and relations between the above-mentioned norms of a solid spherical harmonic (inner and outer) of order n and its spatial gradient. In all cases the gradient's norm differs from the harmonic's norm by a factor of n -th order.

As examples we show relations between the norms of inner solid spherical harmonics

$$r\langle \text{grad}U_n \rangle = n\langle U_n \rangle, \quad r\|\text{grad}U_n\| = \sqrt{n(2n+k-2)}\|U_n\|,$$

and outer ones

$$r\langle \text{grad}V_n \rangle = (n+k-2)\langle V_n \rangle, \quad r\|\text{grad}V_n\| = \sqrt{(n+k-2)(2n+k-2)}\|V_n\|.$$

The symbols $\langle \cdot \rangle$ and $\|\cdot\|$ designate the Chebyshev's and Euclidean norms, respectively.

Bibliogr. 4 references.

UDC 519.67

Grigoriev M. I., Malozemov V. N., Sergeev A. N. **On the classification of rational second-order Bezier curves** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 103–108.

Rational (projective) Bezier curves are determined by three points on a plane and by positive weights ascribed to these points. It is known that such curves are either an arc of a parabola or an arc of an ellipse or an arc of a hyperbola.

An equation for projective Bezier curves in barycentric coordinates is deduced in this paper. This equation depends on a parameter. A complete classification of considered curves depending on the parameter's values is given.

Bibliogr. 4 references.

UDC 517.938

Zuber I. E., Gelig A. Kh. **Robust invariant stabilization of nonlinear continuous and discrete systems** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 109–115.

Two classes of continuous systems designed by differential equations, where its coefficients are function of state vector are considered. These systems have two scalar controls and constant acting scalar disturbance.

The analytical synthesis of such a control is performed that considered system has property of invariantness: scalar output of system strives for zero with time striving for infinity and does not depend on constant acting disturbance, the upper limit of state vector norm is evaluated by upper limit of disturbance norm. The case when coefficients of system are undergone by noncontrolled additive disturbance is considered. In this case an evaluation of upper limit of output norm is received as known function of value of this disturbance. The worked out method of synthesis is based on constructing the Lyapunov function as positive definite quadratic form with the Jacobi matrix.

The some problem is solved for discrete system with analogical structure. In this case the Lyapunov function is constructed as quadratic form with diagonal positive matrix.

Bibliogr. 10 references.

UDC 517.946

Videman J. H., Nazarov S. A. **The refined non-linear Reynolds' equation for a thin flow of a viscous incompressible fluid** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 97–102.

Based on an asymptotic analysis of the Navier—Stokes equations, a scalar nonlinear second-order equation is constructed to describe a liquid flow between two closely placed rigid walls. This equation, inheriting the convective term and the surface curvature, is a refinement of the Reynolds' equation and delivers two-term asymptotics of the solution to the initial problem. Integral and point-wise error estimates are obtained and it is demonstrated that any further improvement of one-dimensional model is impossible.

Bibliogr. 11 references.

UDC 539.3

Gasratova N. A., Shamina V. A. **Solution in terms of stresses for linear axisymmetric problem for sphere and elastic space with spherical cavity** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 122–128.

In linear theory of elasticity it is possible to solve problems in displacements or stresses. The second approach is preferable when displacements are not required. But this approach is convenient to use if the static boundary conditions as well as cinematic ones are formulated in stresses. It is possible for axisymmetric and plane problems and also for problems of linear shell theory. Therefore the basic equations in terms of stresses are used to investigate the axisymmetric stress-strain state of sphere and elastic space with spherical cavity. Analogous problem is considered with the Boussinesq stress functions by W. Nowacki. The solution is presented as series of spherical functions and r — radial coordinate power. In the present paper the solution is expressed in the form of $\cos \theta$ power series. The series coefficients depending on r are defined from the system of differential equations that are reduced to the form that suitable for integrating. The derived relations may be useful to solve the axisymmetric problems in which boundary conditions are given at the close to sphere surfaces.

Bibliogr. 5 references.

UDC 531.011

Almazova S. V. **Determination of a minimal number of parameters defining a spectrum of natural frequencies of a certain mechanical system** // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 116–121.

Application of the approximation method suggested by S. A. Zegzhda and M. P. Yushkov for determining natural frequencies and natural modes of elastic body system in terms of natural frequencies and natural modes of the bodies, which the system consists of, is considered. The method is used for studying a three-beam system, in which two beams execute lateral vibration and one beam executes longitudinal vibration. In the system there is a spring connecting two beams. According to the method the solution is defined in redundant coordinates, which are subject to the holonomic constraints describing conditions of connection of the beams to each other. For determining natural frequencies of the three-beam system considered the determinant, the order of which is equal to the number of holonomic constraints, is obtained. The elements of the determinant are expressed by infinite series. For their approximation calculating the finite number of first summands is considered dynamically, and the rest infinite number of summands is considered quasistatically with the help of formulas of strength of materials. For calculation the determinant elements are represented

in dimensionless form, the least number of dimensionless parameters determining the frequency spectrum of the mechanical system considered appears to be equal to eight.

Bibliogr. 4 references.

UDC 539.3

Lukin A. A., Morozov V. A., Sudenkov Yu. V. Fracture of solid-state dielectrics under conditions of a near-surface electrical breakdown of submicrosecond duration // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 133–140.

By the example of one of representatives of rocks the contribution to fracture by electroimpulse mode of various sources, the basic of which appeared to be thermoelastic stress, is estimated. The dependence of the number of exposures of acting electroimpulse of different amplitude on the breaking stress is constructed and the dynamic threshold of fracture is estimated.

Bibliogr. 9 references.

UDC 514.8:534.1:531.36

Pasynkova I. A., Stepanova P. P. On influence of the massive compliant supports on the critical frequencies of an unbalanced Jeffcott rotor // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 141–147.

This paper deals with the dynamics of an unbalanced Jeffcott rotor with 4 d.o.f., mounted in the massive compliant supports. The rotor is assumed to be a dynamically symmetrical rigid body attached to a massless linearly elastic shaft. The rotor is made to rotate by a motor which is capable of maintaining a constant angular velocity. Bearings are assumed to be point masses on the shaft's ends. The elastic supports possess central symmetry. Damping is neglected.

It has been shown that the forward synchronous precession is hyperboloidal. Self-centering of the rotor under sufficiently high frequencies does not occur. The equation of the fourth order with respect to square of critical frequency has been obtained. It has been shown that it always has three positive roots in case of a dynamically oblate rotor and four positive roots in case of a dynamically prolate rotor. Influence of masses and elasticity of the mountings results in shift of critical frequencies to the lower part of spectrum and appearance of two additional critical frequencies connected with dynamic properties of supports. The critical frequencies of the turbomolecular pump have been found and compared with the known frequencies in case of massless and rigid bearings.

Bibliogr. 9 references.

UDC 539.3

Krakovskaya E. V. On the deformation of a connected spherical shell under the internal pressure // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 129–132.

The article contains the comparative analysis of the analytical and numerical solutions for the problem of connected eye shells (sclera and lamina cribrosa) deformations under the normal pressure. The lamina cribrosa is the part of a sclera, which is weakened by a system of pores. According to the experimental data the site of damage to nerve fibers under glaucoma is just the scleral lamina cribrosa.

An eye shell is modeled as two connected isotropic or transverse-isotropic shells, the sclera and the lamina cribrosa, that is assumed to be a shallow shell of the same radius as the sclera.

The effect of increase of the intraocular pressure on the diameter of the sclera ring and also of the spherical shell deformation on the deflection of the lamina cribrosa are studied.

The analytical solution is obtained by means of the asymptotic integration method by V. Novozhilov and complex equations method by A. Nazarov. Six continuity conditions for the moments, strains and angles of the shell normal rotation on the shell connection line are used. The numerical solution for a three-dimensional mathematical model is obtained by means of the FEM package ANSYS.

The comparison of the analytical and numerical solutions shows that the values of the maximal deflection for the lamina cribrosa differ for 6%, and the change of the size of a scleral canal is much less than the deflections and therefore may be neglected.

Moreover it is revealed in the analysis of the isotropic and transverse-isotropic shells that the anisotropy sufficiently effects both the size and the shape of deformations in the lamina cribrosa. The deflection of the transverse-isotropic spherical shell is smaller and the shape of deformation represents better the actual deformations of the sclera and the lamina cribrosa.

Bibliogr. 6 references.

UDC 524.3/4-32

Ossipkov L. P. A model for corona of a spherical star cluster // Vestnik St.Petersburg University. Ser. 1. 2008. Issue 2. P. 148-157.

A general model for outer layers of steady spherical star systems is considered. It is supposed that it is formed by stars flying out of a system along ‘ballistic’ orbits. Expressions for density runs and velocity dispersions are found. Models of three kinds are studied, namely, systems with an isotropic velocity distribution, spheres with purely radial orbits, and models with an ellipsoidal velocity distribution. The following isotropic distribution functions were considered as examples: the truncated Maxwellian distribution, King’s model, and the spherical analog of Perek’s model. The truncated Maxwellian distribution of radial velocities was an example of radial orbit systems. The truncated Schwarzschildian velocity distribution and generalized polytropes by Kuzmin and Veltmann were considered as examples of models with ellipsoidal velocity distribution. Neglecting self-gravitation of coronas it was possible to find density asymptotics for all models.

Bibliogr. 34 references.