Sums of a Special Class of Generalized Stoynov Distributions[1]

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Generalized Stoynov distributions are weighted versions of generalized Laplace distributions. By analogy with generalized Laplace distributions, two kinds of generalized Stoynov distributions are considered, one-sided and two-sided. The formulas for the sum of a special class of generalized Stoynov distributions are presented.

Key words: Laplace distribution, Stoynov distribution, generalized Laplace distribution, generalized Stoynov distribution.

1. Generalized Stoynov Distributions

Generalized Stoynov distribution can be considered as weighted generalized Laplace distribution by a polynomial weight function. We define generalized (double-side) Stoynov distribution as a distribution of random variable ξ with probability density function given by the equation

$$f_\xi(x) = C_D(\mu,\sigma,k,m,s,p)e^{-(\frac{|x-\mu|}{\sigma})^k}(1+|\frac{x-\mu}{s}|^p)^m, \quad \mu \in R, \sigma \in R^+, p \in N, m \in N, k \in N \tag{1}$$

We will denote this distribution as $\xi \in GSD(\mu,\sigma,k,m,s,p)$.

We define generalized (one-side) Stoynov distribution with normalized weight argument as a distribution of random variable ξ with probability density function given by the equation

$$f_\xi(x) = C_O(\mu,\sigma,k,m,s,p)e^{-(\frac{x-\mu}{\sigma})^k}(1+(\frac{x-\mu}{s})^p)^m, \quad x \ge \mu, \mu \in R, \sigma \in R^+, p \in N, m \in N, k \in N \tag{2}$$

We will denote this distribution as $\xi \in GSO(\mu,\sigma,k,m,s,p)$.

2. Some Special Cases of Generalized Stoynov Distributions

It is clear that generalized Laplace distributions are special case of generalized Stoynov distribution. We have $GSO(\mu, \sigma, k, 0, s, p) = GLO(\mu, \sigma, k)$ and $GSD(\mu, \sigma, k, 0, s, p) = GLO(\mu, \sigma, k)$. The special case $\xi \in GSO(0, \lambda^{-1}, 1, 1, 1, 1)$ is the distribution of Lindley. The special case $\xi \in GSO(0, \lambda^{-1}, 2, 1, 1, 1)$ is the distribution of Stoynov described in [1]. In this case $C_O(0, \lambda^{-1}, 2, 1, 1, 1) = \frac{\lambda^3}{\lambda^2+2\lambda+2}$.

3. Sum of Two $GSO(0, \sigma, 1, m, 1, 1)$ Distributions

Here we will consider sum of two $GSO(0, \sigma, 1, m, 1, 1)$ distributions. Let $\xi \in GSO(0, \sigma_1, 1, m_1, 1, 1)$ and $\eta \in GSO(0, \sigma_2, 1, m_2, 1, 1)$. We will calculate the density of the sum $\phi = \xi + \eta$. We have

$$f_\xi(x) = C_O(0, \sigma_1, 1, m_1, 1, 1)e^{\frac{-x}{\sigma_1}}(1+x)^{m_1} = \tag{3}$$

$$= \frac{1}{\sigma_1 \sum_{k=1}^{m_1} k! \begin{pmatrix} m_1 \\ k \end{pmatrix} \sigma_1^k} e^{\frac{-x}{\sigma_1}}(1+x)^{m_1}, x > 0,$$

$$f_\xi(x) = 0, x < 0$$

and

$$f_\eta(x) = C_O(0, \sigma_2, 1, m_2, 1, 1)e^{\frac{-x}{\sigma_2}}(1+x)^{m_2} = \tag{4}$$

$$= \frac{1}{\sigma_2 \sum_{k=1}^{m_2} k! \begin{pmatrix} m_2 \\ k \end{pmatrix} \sigma_2^k} e^{\frac{-x}{\sigma_2}}(1+x)^{m_2}, x > 0$$

$$f_\eta(x) = 0, x < 0.$$

We have

$$f_\phi(u) = \frac{1}{\sigma_1 \sum_{k=1}^{m_1} k! \begin{pmatrix} m_1 \\ k \end{pmatrix} \sigma_1^k} \frac{1}{\sigma_2 \sum_{k=1}^{m_2} k! \begin{pmatrix} m_2 \\ k \end{pmatrix} \sigma_2^k} *$$

$$* e^{\frac{-u}{\sigma_1}} \int_0^u e^{v\left(\frac{1}{\sigma_1} - \frac{1}{\sigma_2}\right)} (1+u-v)^{m_1}(1+v)^{m_2} dv =$$

$$= \frac{1}{\sigma_1 \sum_{k=1}^{m_1} k! \begin{pmatrix} m_1 \\ k \end{pmatrix} \sigma_1^k} \frac{1}{\sigma_2 \sum_{k=1}^{m_2} k! \begin{pmatrix} m_2 \\ k \end{pmatrix} \sigma_2^k} e^{\frac{-u}{\sigma_1}} I,\ u > 0, \tag{5}$$

where

$$I = \int_0^u e^{v\left(\frac{1}{\sigma_1}-\frac{1}{\sigma_2}\right)}(1+u-v)^{m_1}(1+v)^{m_2}dv. \qquad (6)$$

Now we will calculate the integral I. We will consider the case where $\sigma_1 = \sigma_2 = \sigma$. In this case,

$$\begin{aligned} I = \int_0^u e^{v\left(\frac{1}{\sigma_1}-\frac{1}{\sigma_2}\right)}(1+u-v)^{m_1}(1+v)^{m_2}dv = \\ = \int_0^u (1+u-v)^{m_1}(1+v)^{m_2}dv = \\ = \int_0^u \sum_{k=1}^{m_1} \left(\begin{array}{c} m_1 \\ k \end{array} \right)(1+u)^k(-1)^{m_1-k}v^{m_1-k} * \\ * \sum_{s=1}^{m_2} \left(\begin{array}{c} m_2 \\ s \end{array} \right) v^s dv = \\ = \sum_{k=1}^{m_1} \left(\begin{array}{c} m_1 \\ k \end{array} \right)(1+u)^k(-1)^{m_1-k} * \\ * \sum_{s=1}^{m_2} \left(\begin{array}{c} m_2 \\ s \end{array} \right) \int_0^u v^{m_1-k+s} dv = \\ = \sum_{k=1}^{m_1} \left(\begin{array}{c} m_1 \\ k \end{array} \right)(1+u)^k(-1)^{m_1-k} * \\ * \sum_{s=1}^{m_2} \left(\begin{array}{c} m_2 \\ s \end{array} \right) \frac{v^{m_1-k+s+1}}{m_1-k+s+1}|_0^u = \\ = \sum_{k=1}^{m_1} \left(\begin{array}{c} m_1 \\ k \end{array} \right)(1+u)^k(-1)^{m_1-k} * \\ * \sum_{s=1}^{m_2} \left(\begin{array}{c} m_2 \\ s \end{array} \right) \frac{u^{m_1-k+s+1}}{m_1-k+s+1}. \end{aligned} \qquad (7)$$

In the case when $\xi \in GSO(0,\sigma,1,2,1,1)$ and $\eta \in GSO(0,\sigma,1,2,1,1)$ we have sum of Stoynov distributions. In this case,

$$f_\xi(x) = f_\eta(x) = C_O(0,\sigma,1,2,1,1)e^{\frac{-x}{\sigma}}(1+x)^2 = \qquad (8)$$

$$= \frac{1}{\sigma(1+\sigma+\sigma^2)}e^{\frac{-x}{\sigma}}(1+x)^2, x > 0$$

$$f_\xi(x) = f_\eta(x) = 0, x < 0.$$

Next,

$$\begin{aligned} f_\phi(u) &= \frac{1}{\sigma^2(1+\sigma+\sigma^2)^2} e^{\frac{-u}{\sigma}} * \\ &* \int_0^u (1+u-v)^2(1+v)^2 dv = \\ &= \frac{1}{\sigma^2(1+\sigma+\sigma^2)^2} e^{\frac{-u}{\sigma}} I,\ u > 0 \end{aligned} \tag{9}$$

where

$$\begin{aligned} I &= \int_0^u (1+u-v)^2(1+v)^2 dv = \\ &= \frac{30u + 60u^2 + 40u^3 + 10u^4 + u^5}{30}. \end{aligned} \tag{10}$$

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Список литературы

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