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Guaranteed cost control of neural networks with various activation functions and mixed time-varying delays in state and control

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Abstract

This article studies a guaranteed cost control problem for a class of neural networks with various activation functions and mixed time-varying delays in state and control. Attention is focused on the design of memory feedback controller such that the resulting closed-loop system is exponentially stable and an adequate level of performance is also guaranteed. Using the Lyapunov method and linear matrix inequality technique, a criteria for the existence of the controller are derived in terms of LMIs. A numerical example is included to illustrate the effectiveness of our results.

Keywords: Cellular neutral networks; guaranteed cost control; mixed delay; Lyapunov function; Linear matrix inequalities (LMIs).
2000MSC: 34D05, 34D20, 34K20, 34K35

1 Introduction

Cellular neural networks with time-varying delays have been extensively studied over the past two decades and have found many application in variety of areas such as signal processing, pattern recognition, static image processing, associate memory, and combinatorial optimization. Since time delay effects are often a source of instability and poor performance of the neural networks, the problem of stability analysis or stabilization of neural networks with time-varying delays has attracted many researcher [5, 6, 8, 12, 13]. On the other hand, in many practical system, it is desirable to design the control system which is not only stable but also guarantee an adequate level of performance. One approach to this problem is the so-called guaranteed cost control approach first introduced by Chang and Peng [2]. This approach has the advantage of providing an upper bound on a given performance index and thus the system performance degradation incurred by time delays is guaranteed to be less than this bound. Base on this idea, some results have been proposed for discrete-time with constant delays systems [3, 10, 11], and for continuous-time with constant delays systems [2, 7, 9]. To the best of our knowledge, so far, no result on the guaranteed cost control of the neural networks with mixed time-varying delay in state and control is available in literature. This motivates our present investigation.

In this paper, we consider the problem of guaranteed cost control of neural networks with various activation functions and mixed time-varying delays in state and control. The novel feature of the results obtained in this paper is twofold. First, the system considered in this paper is mixed time-varying delays in state and control. Second, by using improved Lyapunov-Krasovskii functionals combined with LMIs technique, a delay-dependent criterion for existence of the guaranteed cost controller is derived in terms of LMIs. The approach also allows to compute simultaneously the two bounds the characterize the exponential stability rate of the solution and design a upper bound of cost function for the system.

The paper is organized as follows. Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results. Guaranteed cost control of neural networks with various activation functions and mixed time-varying delays in state and control are presented in Section 3. Numerical examples showing the feasibility and effectiveness of the conditions are given in Section 4. The paper ends with conclusions and cited references.

2 Preliminaries

The following notation will be used in this paper: $\mathbb{R}^+$ denotes the set of all real non-negative numbers; $\mathbb{R}^n$ denotes the $n-$dimensional space with the scalar product $\langle .,. \rangle$ and the vector norm $\| \, . \, \|$; $\mathbb{R}^{n\times r}$ denotes the space of all matrices of $(n \times r)-$dimensions; A^T denotes the transpose of matrix A; A is symmetric if

$A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\lambda; \lambda \in \lambda(A)\}$.; $x_t := \{x(t+s) : s \in [-h, 0]\}, \| x_t \|= \sup_{s\in[-h,0]} \| x(t+s) \|$; $C([0,t], \mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n-$valued continuous functions on $[0,t]$; $L_2([0,t], \mathbb{R}^m)$ denotes the set of all the $\mathbb{R}^m-$valued square integrable functions on $[0,t]$; Matrix A is called semi-positive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in \mathbb{R}^n; A$ is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0; A > B$ means $A - B > 0$. The notation diag$\{\ldots\}$ stands for a block-diagonal matrix. The symmetric term in a matrix is denoted by $*$.

Consider the following cellular neural networks with mixed time-varying delays in state and control of the form

$$\begin{aligned} \dot{x}(t) = &-Ax(t) + W_0 f(x(t)) + W_1 g(x(t-\tau_1(t))) + W_2 \int_{t-\tau_2(t)}^{t} h(x(s))\,ds \\ &+ B_0 u(t) + B_1 u(t-\tau_3(t)) + B_2 \int_{t-\tau_4(t)}^{t} u(s)\,ds \\ x(t) = &\ \phi(t), t \in [-d, 0], \quad d = \max\{\tau_1, \tau_2, \tau_3, \tau_4\}, \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector of the neural networks; $u(t) \in L_2([0,s], \mathbb{R}^m), \forall s > 0$, is the control input vector of the neural networks; n is the number of neurals, and

$$\begin{aligned} f(x(t)) &= [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T, \\ g(x(t)) &= [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T, \\ h(x(t)) &= [h_1(x_1(t)), h_2(x_2(t)), \ldots, h_n(x_n(t))]^T \end{aligned}$$

are the neural activation functions. The diagonal matrix $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$ represents the self-feedback term and $W_0, W_1, W_2, B_0, B_1, B_2$ are given real constant matrices with appropriate dimensions. The time-varying delay functions $\tau_1(t), \tau_2(t), \tau_3(t), \tau_4(t)$ satisfy the condition

$$\begin{aligned} &0 \leq \tau_1(t) \leq \tau_1, \quad \dot{\tau}_1(t) \leq \delta_1 < 1, \\ &0 \leq \tau_2(t) \leq \tau_2, \\ &0 \leq \tau_3(t) \leq \tau_3, \quad \dot{\tau}_3(t) \leq \delta_2 < 1, \\ &0 \leq \tau_4(t) \leq \tau_4. \end{aligned}$$

The initial functions $\phi(t) \in C([-d, 0], \mathbb{R}^n), d = \max\{\tau_1, \tau_2, \tau_3, \tau_4\}$, with the uniform norm $\| \phi \|= \max_{t\in[-d,0]} \| \phi(t) \|$. We assume that the activation functions $f(x), g(x), h(x)$ are Lipschitz with the Lipschitz constains $f_i, g_i, h_i > 0$:

$$\begin{aligned} &\mid f_i(\xi_1) - f_i(\xi_2) \mid \leq f_i \mid \xi_1 - \xi_2 \mid, \quad i = 1, 2, \dots, n, \forall \xi_1, \xi_2 \in R, \\ &\mid g_i(\xi_1) - g_i(\xi_2) \mid \leq g_i \mid \xi_1 - \xi_2 \mid, \quad i = 1, 2, \dots, n, \forall \xi_1, \xi_2 \in R, \\ &\mid h_i(\xi_1) - h_i(\xi_2) \mid \leq h_i \mid \xi_1 - \xi_2 \mid, \quad i = 1, 2, \dots, n, \forall \xi_1, \xi_2 \in R. \end{aligned} \tag{2}$$

Definition 2.1 Given $\alpha > 0$. The system (1) is $\alpha-$exponentially stable if there exist a positive number $\gamma > 0$ such that every solution $x(t, \phi)$ satisfies the following condition:

$$\| x(t, \phi) \| \leq \gamma e^{-\alpha t} \| \phi \|, \quad \forall t \geq 0.$$

Definition 2.2 Given $\alpha > 0$. The system (1) is globally $\alpha-$exponentially stabilizable if there is a feedback control $u(t) = Kx(t)$, such that the closed-loop time-delay system

$$\begin{aligned} \dot{x}(t) &= -[A_0 - B_0 K]x(t) + W_0 f(x(t)) + W_1 g(x(t - \tau_1(t))) + B_1 K x(t - \tau_3(t)) \\ &\quad + W_2 \int_{t-\tau_2(t)}^{t} h(x(s))\, ds + B_2 \int_{t-\tau_4(t)}^{t} K x(s)\, ds \\ x(t) &= \phi(t), t \in [-d, 0], \quad d = \max\{\tau_1, \tau_2, \tau_3, \tau_4\} \end{aligned} \tag{3}$$

is $\alpha-$exponentially stable.
Associated with the system (1) is the cost function

$$J = \int_0^\infty [x^T(t) M_1 x(t) + u^T(t) M_2 u(t)]\, dt, \tag{4}$$

where $M_1 \in \mathbb{R}^{n \times n}$ and $M_2 \in \mathbb{R}^{m \times m}$ are given symmetric positive-definite matrices.

Here, the objective of this article is to develop a procedure to design a memory state feedback controller $u(t)$ for the system (1) and cost function (4) such that the resulting closed-loop system is $\alpha-$exponentially stable and the closed-loop value of the cost function (4) satisfies $J \leq J^*$, where J^* is some specified constant.

Definition 2.3 For the system (1) and cost function (4), if there exist a control law $u^*(t)$ and a positive J^* such that for all admissible delays, the system (1) is $\alpha-$exponentially stable and the closed-loop value of the cost function (4) satisfies $J \leq J^*$, then J^* is said to be a guaranteed cost and $u^*(t)$ is said to be a guaranteed cost control law of the system (1) and cost function (4).

We introduce the following technical well-known propositions, which will be used in the proof of our results.

Proposition 2.1 Let P, Q be matrices of appropriate dimensions and Q is symmetric positive definite. Then

$$2\langle Py, x\rangle - \langle Qy, y\rangle \leq \langle PQ^{-1}P^T x, x\rangle, \quad \forall (x, y).$$

Proposition 2.2 (Gu, 2000). For any symmetric positive definite matrix $M > 0$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds

$$\Big(\int_0^\gamma \omega(s)\, ds\Big)^T M \Big(\int_0^\gamma \omega(s)\, ds\Big) \leq \gamma \Big(\int_0^\gamma \omega^T(s) M \omega(s)\, ds\Big)$$

Proposition 2.3 (Schur complement lemma). Given constant symmetric matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

3 Main result

Let us denote

$$\begin{aligned} \Sigma = &-AP - PA^T - (B_0 Y + Y^T B_0^T) + 2\alpha P + W_0 D_0 W_0^T \\ &+ (1-\delta_1)^{-1} e^{2\alpha\tau_1} W_1 D_1 W_1^T \\ &+ (1-\delta_2)^{-1} e^{2\alpha\tau_3} B_1 B_1^T + \tau_2 e^{2\alpha\tau_2} W_2 D_2 W_2^T + \tau_4 e^{2\alpha\tau_4} B_2 B_2^T, \\ G = &\operatorname{diag}\{g_i, i = 1, 2, \ldots, n\}, \quad H = \operatorname{diag}\{h_i, i = 1, 2, \ldots, n\}, \\ F = &\operatorname{diag}\{f_i, i = 1, 2, \ldots, n,\} \end{aligned}$$

$$g^2 = \max\{g_i^2, i = 1, 2, \ldots, n\}, \quad h^2 = \max\{h_i^2, i = 1, 2, \ldots, n\}, K = -YP^{-1},$$

$$\lambda_1 = \lambda_{\min}(P^{-1}),$$

$$\lambda_2 = \lambda_{\max}(P^{-1}) + \lambda_{\max}(D_1^{-1}) g^2 \tau_1 + \lambda_{\max}(D_2^{-1}) h^2 \tau_2^2 + (\tau_3 + \frac{1}{2}\tau_4^2)\lambda_{\max}(K^T K)$$

Theorem 3.1. *For given* $\alpha > 0, M_1 > 0$ *and* $M_2 > 0, u(t) = -YP^{-1}x(t)$ *is a guaranteed cost controller for the system* (1) *if there exists a symmetric positive definite matrix* $P \in \mathbb{R}^{n\times n}$*, three diagonal positive matrices* $D_i, i = 0, 1, 2$ *and a matrix* Y *with appropriate dimension such that the following LMI holds:*

$$\Xi = \begin{bmatrix} \Sigma & PF & PG & \tau_2 PH & PM_1 & Y^T M_2 & (1+\tau_4)Y^T \\ FP & -D_0 & 0 & 0 & 0 & 0 & 0 \\ GP & 0 & -D_1 & 0 & 0 & 0 & 0 \\ \tau_2 HP & 0 & 0 & -D_2 & 0 & 0 & 0 \\ M_1 P & 0 & 0 & 0 & -M_1 & 0 & 0 \\ M_2 Y & 0 & 0 & 0 & 0 & -M_2 & 0 \\ (1+\tau_4)Y & 0 & 0 & 0 & 0 & 0 & -(1+\tau_4)I_m \end{bmatrix} < 0. \quad (5)$$

Then, the upper bound of cost function for the system (2.1) *is*

$$J \leq J^* = \lambda_2 \parallel \phi \parallel^2 .$$

Proof. Let us denote $X = P^{-1}$. With the feedback control $u(t) = -YP^{-1}$, we consider the Lyapunov-Krasovskii functional for closed-loop system

$$V(t, x_t) = \sum_{i=1}^{5} V_i(t, x_t),$$

where

$$V_1 = x^T(t)Xx(t),$$
$$V_2 = \int_{t-\tau_1(t)}^{t} e^{2\alpha(s-t)} g^T(x(s)) D_1^{-1} g(x(s))\, ds,$$
$$V_3 = \int_{-\tau_2}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)} h^T(x(\theta)) D_2^{-1} h(x(\theta))\, d\theta\, ds,$$
$$V_4 = \int_{t-\tau_3(t)}^{t} e^{2\alpha(s-t)} x^T(s) K^T K x(s)\, ds,$$
$$V_5 = \int_{-\tau_4}^{0} \int_{t+s}^{t} e^{2\alpha(\theta-t)} x^T(\theta) K^T K x(\theta)\, d\theta\, ds.$$

It is easy to check that

$$\lambda_1 \parallel x(t) \parallel^2 \leq V(t, x_t) \leq \lambda_2 \parallel x_t \parallel^2, \quad t \in \mathbb{R}^+. \quad (6)$$

Taking derivative of V_1 along solutions of the closed-loop system (3), we get

$$\begin{aligned}\dot{V}_1 =& x^T(t)[-XA - A^TX - X(B_0Y + Y^TB_0^T)X]x(t)\\ &+ 2x^T(t)XW_0f(x(t)) + 2x^T(t)XW_1g(x(t-\tau_1(t)))\\ &+ 2x^T(t)XB_1u(t-\tau_3(t))\\ &+ 2x^T(t)XW_2\int_{t-\tau_2(t)}^{t} h(x(s))\,ds + 2x^T(t)XB_2\int_{t-\tau_4(t)}^{t} u(s)\,ds\end{aligned}$$

Applying Proposition 2.1 and Proposition 2.2 gives

$$2x^T(t)XW_0f(x(t)) \le x^T(t)XW_0D_0W_0^TXx(t) + f^T(x(t))D_0^{-1}f(x(t));$$

$$\begin{aligned}2x^T(t)XW_1g(x(t-\tau_1(t)) \le& (1-\delta_1)^{-1}e^{2\alpha\tau_1}x^T(t)XW_1D_1W_1^TXx(t)\\ &+ (1-\delta_1)e^{-2\alpha\tau_1}g(x(t-\tau_1(t)))^TD_1^{-1}g(x(t-\tau_1(t)));\end{aligned}$$

$$\begin{aligned}2x^T(t)XB_1u(t-\tau_3(t)) \le& (1-\delta_2)^{-1}e^{2\alpha\tau_3}x^T(t)XB_1B_1^TXx(t)\\ &+ (1-\delta_2)e^{-2\alpha\tau_3} \parallel u(t-\tau_3(t)) \parallel^2;\end{aligned}$$

$$\begin{aligned}&2x^T(t)XW_2\int_{t-\tau_2(t)}^{t} h(x(s))\,ds \le \tau_2e^{2\alpha\tau_2}x^T(t)XW_2D_2W_2^TXx(t)\\ &+ \frac{1}{\tau_2}e^{-2\alpha\tau_2}\Big(\int_{t-\tau_2(t)}^{t} h(x(s))\,ds\Big)^TD_2^{-1}\Big(\int_{t-\tau_2(t)}^{t} h(x(s))\,ds\Big)\\ &\le \tau_2e^{2\alpha\tau_2}x^T(t)XW_2D_2W_2^TXx(t) + e^{-2\alpha\tau_2}\int_{t-\tau_2(t)}^{t} h^T(x(s))D_2^{-1}h(x(s))\,ds\\ &\le \tau_2e^{2\alpha\tau_2}x^T(t)XW_2D_2W_2^TXx(t) + e^{-2\alpha\tau_2}\int_{t-\tau_2}^{t} h^T(x(s))D_2^{-1}h(x(s))\,ds;\end{aligned}$$

$$\begin{aligned}&2x^T(t)XB_2\int_{t-\tau_4(t)}^{t} u(s)\,ds \le \tau_4e^{2\alpha\tau_4}x^T(t)XB_2B_2^TXx(t)\\ &+ \frac{1}{\tau_4}e^{-2\alpha\tau_4}\Big(\int_{t-\tau_4(t)}^{t} u(s)\,ds\Big)^T\Big(\int_{t-\tau_4(t)}^{t} u(s)\,ds\Big)\\ &\le \tau_4e^{2\alpha\tau_4}x^T(t)XB_2B_2^TXx(t) + e^{-2\alpha\tau_4}\int_{t-\tau_4(t)}^{t} \parallel u(s) \parallel^2\,ds\\ &\le \tau_4e^{2\alpha\tau_4}x^T(t)XB_2B_2^TXx(t) + e^{-2\alpha\tau_4}\int_{t-\tau_4}^{t} \parallel u(s) \parallel^2\,ds.\end{aligned}$$

Therefor

$$\begin{aligned}\dot{V}_1 \le & x^T(t)[-XA - A^TX - X(B_0Y + Y^TB_0^T)X]x(t)\\ & + x^TX(t)\Big[W_0D_0W_0^T + (1-\delta_1)^{-1}e^{2\alpha\tau_1}W_1D_1W_1^T + (1-\delta_2)^{-1}e^{2\alpha\tau_3}B_1B_1^T\\ & + \tau_2e^{2\alpha\tau_2}W_2D_2W_2^T + \tau_4e^{2\alpha\tau_4}B_2B_2^T\Big]Xx(t)\\ & + f^T(x(t))D_0^{-1}f(x(t)) + (1-\delta_1)e^{-2\alpha\tau_1}g(x(t-\tau_1(t)))^TD_1^{-1}g(x(t-\tau_1(t)))\\ & + (1-\delta_2)e^{-2\alpha\tau_3} \parallel u(t-\tau_3(t)) \parallel^2 + e^{-2\alpha\tau_2}\int_{t-\tau_2}^t h^T(x(s))D_2^{-1}h(x(s))\,ds\\ & + e^{-2\alpha\tau_4}\int_{t-\tau_4}^t \parallel u(s) \parallel^2\ ds\end{aligned} \tag{7}$$

Next, the derivatives of $V_k, k = 2, \ldots, 5$ give

$$\begin{aligned}\dot{V}_2 &\le -2\alpha V_2 + (g(x(t)))^TD_1^{-1}(g(x(t)))\\ &\qquad - (1-\delta_1)e^{-2\alpha\tau_1}(g(x(t-\tau_1(t))))^TD_1^{-1}(g(x(t-\tau_1(t))));\\ \dot{V}_3 &\le -2\alpha V_3 + \tau_2(h(x(t)))^TD_2^{-1}(h(x(t)))\\ &\qquad - e^{-2\alpha\tau_2}\int_{t-\tau_2}^t (h(x(s)))^TD_2^{-1}(h(x(s)))\,ds;\\ \dot{V}_4 &\le -2\alpha V_4 + x^T(t)XY^TYXx(t) - (1-\delta_2)e^{-2\alpha\tau_3} \parallel u(t-\tau_3(t)) \parallel^2;\\ \dot{V}_5 &\le -2\alpha V_5 + \tau_4x^T(t)XY^TYXx(t) - e^{-2\alpha\tau_4}\int_{t-\tau_4}^t \parallel u(s) \parallel^2\ ds.\end{aligned} \tag{8}$$

From (7) – (8), we obtain

$$\begin{aligned}&\dot{V} + 2\alpha V\\ &\le x^T(t)\Big[-XA - A^TX - X(B_0Y + Y^TB_0^T)X + 2\alpha X + XW_0D_0W_0^TX\\ &\quad + (1-\delta_1)^{-1}e^{2\alpha\tau_1}XW_1D_1W_1^TX + (1-\delta_2)^{-1}e^{2\alpha\tau_3}XB_1B_1^TX\\ &\quad + \tau_2e^{2\alpha\tau_2}XW_2D_2W_2^TX + \tau_4e^{2\alpha\tau_4}XB_2B_2^TX + (1+\tau_4)XY^TYX\Big]x(t)\\ &\quad + f^T(x(t))D_0^{-1}f(x(t)) + g^T(x(t))D_1^{-1}g(x(t)) + \tau_2h^T(x(t))D_2^{-1}h(x(t)).\end{aligned} \tag{9}$$

Using the condition (2) and since the matrices $D_i > 0, i = 0, 1, 2$ are diagonal, we have

$$\begin{aligned} &f^T(x(t))D_0^{-1}f(x(t)) \le x^T(t)FD_0^{-1}Fx(t),\\ &g^T(x(t))D_1^{-1}g(x(t)) \le x^T(t)GD_1^{-1}Gx(t),\\ &\tau_2 h^T(x(t))D_2^{-1}h(x(t)) \le \tau_2 x^T(t)HD_2^{-1}Hx(t). \end{aligned} \tag{10}$$

Since (9) and (10), we obtain

$$\dot V + 2\alpha V \le x^T(t)\Omega x(t) - x^T(t)\mathcal{M}x(t),$$

where

$$\begin{aligned} \Omega = &- XA - A^TX - X(B_0Y + Y^TB_0^T)X + 2\alpha X + XW_0D_0W_0^TX\\ &+ (1-\delta_1)^{-1}e^{2\alpha\tau_1}XW_1D_1W_1^TX + (1-\delta_2)^{-1}e^{2\alpha\tau_3}XB_1B_1^TX\\ &+ \tau_2 e^{2\alpha\tau_2}XW_2D_2W_2^TX + \tau_4 e^{2\alpha\tau_4}XB_2B_2^TX + (1+\tau_4)XY^TYX\\ &+ FD_0^{-1}F + GD_1^{-1}G + \tau_2 HD_2^{-1}H + M_1 + XY^TM_2YX, \end{aligned} \tag{11}$$

$$\mathcal{M} = M_1 + XY^TM_2YX.$$

Therefor, if $\Omega < 0$, then

$$\dot V + 2\alpha V \le -x^T(t)\mathcal{M}x(t).$$

Since $\mathcal{M} > 0$, we obtain

$$\dot V + 2\alpha V \le 0, \tag{12}$$

which guarantees the exponentially stabilition of the system by Lyapunov stability theory and the solution $x(t,\phi)$ of the system satisfy

$$\| x(t,\phi) \| \le \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \| \phi \|, \quad \forall t \ge 0.$$

Pre- and post-multiplying both sides of Ω by P, and note that $\Omega < 0$ is equivalent to $P\Omega P < 0$. Applying the Schur complement yield linear matrix inequality (5). Since $V(t,x_t) > 0$, we have

$$\dot V(t,x_t) \le -x^T(t)\mathcal{M}x(t) \tag{13}$$

Integrating both sides of (13) from 0 to s, we obtain

$$V(s,x_s) - V(0,\phi) \le -\int_0^s x^T(t)\mathcal{M}x(t)\,dt.$$

Hence

$$\int_0^s x^T(t)\mathcal{M}x(t)\,dt \le V(0,\phi) = \lambda_2 \parallel \phi \parallel^2 .$$

Given $s \to \infty$, we obtain

$$J = \int_0^\infty x^T(t)\mathcal{M}x(t)\,dt \le \lambda_2 \parallel \phi \parallel^2 = J^*.$$

This completes the proof.

Remark 3.1. *For given $\alpha > 0, M_1 > 0, M_2 > 0$, the criteria for existence of the guaranteed cost control of neural networks with various activation functions and mixed time-varying delays in state and control is derived in terms of LMIs, which can be solve by various efficient covex algorithms [1, 4].*

Remark 3.2. *The neural networks system with various activation functions and mixed time-varying delays considered in previous works (e.g. [5, 6, 8, 12, 13]) are special cases of the system* (1).

4 Numerical examples

Example 4.1. Consider the system (1), where

$$\phi(t) = 5\text{sin}5t, \quad \tau_1(t) = \text{sin}0.5t, \quad \tau_3(t) = \text{sin}0.6t, \tau_4(t) = 0.5|\text{sin}t|,$$

$$\begin{cases} \tau_2(t) = 0.8\sin^2 t & if \quad t \in \mathcal{I} = \cup_{k\ge 0}[2k\pi, (2k+1)\pi] \\ \tau_2(t) = 0 & if \quad t \in R^+ \setminus \mathcal{I}, \end{cases}$$

$$A = \begin{pmatrix} 27 & 0 \\ 0 & 26 \end{pmatrix}, \quad W_0 = \begin{pmatrix} 3 & 0.15 \\ 1 & 2 \end{pmatrix}, \quad W_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad W_2 = \begin{pmatrix} 0.1 & 0.4 \\ 0.5 & 0.2 \end{pmatrix},$$

$$B_0 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad B_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix},$$

$$F = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.4 \end{pmatrix}, \quad G = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.8 \end{pmatrix}, \quad H = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.6 \end{pmatrix}.$$

We see that the time delay functions $\tau_2(t), \tau_4(t)$ are bounded but non-differentiable and $\tau_1 = 1, \tau_2 = 0.8, \tau_3 = 1, \tau_4 = 0.5, \delta_1 = 0.5, \delta_2 = 0.6$.

Given

$$\alpha = 0.5, M_1 = \begin{pmatrix} 3 & 1 \\ 1 & 4 \end{pmatrix}, \quad M_2 = [9],$$

using MATLABs LMI Toolbox, the LMI (5) is feasible with the following matrices:

$$P = \begin{pmatrix} 3.6537 & -0.5970 \\ -0.5970 & 4.1381 \end{pmatrix}, D_0 = \begin{pmatrix} 4.4254 & 0 \\ 0 & 4.4794 \end{pmatrix}, D_1 = \begin{pmatrix} 3.5888 & 0 \\ 0 & 8.6180 \end{pmatrix},$$

$$D_2 = \begin{pmatrix} 17.5413 & 0 \\ 0 & 24.7486 \end{pmatrix}, Y = \begin{bmatrix} -0.0023 & 0.1878 \end{bmatrix}.$$

and accordingly the feedback control is $u(t) = \begin{bmatrix} -0.0069 & -0.0464 \end{bmatrix} x(t)$. Moreover, the solution of closed-loop system satisfy

$$\| x(t, \phi) \| \leq 2.2760 e^{-0.5t} \| \phi \|, \quad \forall t \geq 0,$$

and the optimal guaranteed cost of the closed-loop system is as follows:

$$J \leq J^* = 11.38.$$

5 Conclusions

In this paper, we have presented a solution to the guaranteed cost control problem for a class of neural networks with various activation functions and mixed time-varying delays in state and control in a LMI framework. The existence condition for guaranteed cost memory feedback controllers has been derived in terms of linear matrix inequalities which allows to compute simultaneously the two bounds that characterize the exponential stability of the solution of closed-loop system and design the upper bound of cost function for the system. A numerical example is given to show the effectiveness of the obtained result.

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