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Control in non-linear systems

Synthesis of Controller for Vector Plant, Based on Integral Adaptation Method for Disturbance Suppression

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Abstract. The problem of control over a nonlinear plant on the target manifold with two types of uncertainties in the description is discussed: deterministic disturbances in the right-hand side of the system of differential/difference equations as the unknown functions of time and stochastic disturbances with a zero mean and limited dispersion in the right-hand side of the system of difference equations. The method of integral adaptation is used, which relies on an analytical description of the target attractors – invariant manifolds and introduction of the integrators into the control channels, which make it possible to synthesize adaptive systems without acquiring the current data on the changes of the plant and environment parameters. A search for control in the case of stochastic disturbances is performed from among the strategies minimizing the dispersion of the output macrovariable. The examples and results of the numerical testing of the proposed algorithms are presented for an applied problem of control over the motion of an plant with an immobile center of mass. The presented results can be relevant in all control systems of robotic arms, functioning under the conditions of uncertainty, and in designing the decision-making support systems in controlling compound plants.

Keywords: principle of least action, analytical design of aggregated regulators, nonlinear plant, method of integral adaptation, object with fixed center of mass, random and deterministic disturbances.

1. Introduction

The method of integral adaptation on invariant manifolds has demonstrated its high performance in solving the problems of controlling the complex (nonlinear, multi-loop) dynamic plants [1]. These problems present essential practical challenges for common methods of adaptive control [2-7], since the observer dynamics has to be faster compared to that of changes of the coordinates and variation of the external disturbances.

A conceptual difference of the method of integral adaptation [1, 8-10] from the well-known classical methods of the adaptive control theory consists in the mechanism of introduction of the attractors – invariant manifolds (chaotic manifolds among them) from the synergetic control theory [1, 8] and "backstepping" [11, 12].

In this study we continue solving the issue of applicability of the method of integral adaptation (see more detailed reviews in [1, 8, 9]) to the models with random and deterministic uncertainties in their description .

In Sections 2.1 and 3.1, we discuss the problem of control of a nonlinear vector plant $\mathbf{x} = \left(\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}}\right)^{\mathrm{T}}, \mathbf{x}_i \in R^3, i = 1,2$ using the following description of the dynamics of its coordinates:

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{f}_1\left(\mathbf{x}_1, \mathbf{x}_2\right), \\ \dot{\mathbf{x}}_2 &= \mathbf{f}_2\left(\mathbf{x}_1, \mathbf{x}_2\right) + \boldsymbol{\xi} + \mathbf{u}, \end{aligned} \tag{1.1}$$

where $\left(\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}}\right)^{\mathrm{T}}, \mathbf{x}_i \in R^3, i = 1,2$ is the state vector, $\mathbf{u} \in R^3$ is the control vector, $\mathbf{f}_i \in R^3$ are the nonlinear vector-functions, i=1,2; $\boldsymbol{\xi} \in R^3$ is the unknown limited continuous deterministic disturbance. Recall that manipulator systems (such as robotic arms [13-15]) obey this description; the importance of controlling them under the conditions of noise hardly requires elaboration (e.g., [16-18]).

Sections 2.2 and 3.2 provide the problem statement and describe the controller design for a discrete system obtained from (1.1) relying on the Euler discretization and containing a deterministic uncertainty in its description.

Section 4 focuses on the problem of nonlinear discrete-time vector control over a stochastic plant.

Sections 5 and 6 present the results of a comparative simulation of two systems of control in the model and off-design conditions for the problem of control over a robotic arm.

The design of control systems on the principles of manifolds (see reviews in [9, 19-21]) possesses a number of important properties, for instance: the problem of synthesizing an optimal asymptotic stable control system can be solved without using the Lyapunov functions [8, 9]; a certain correspondence to the physical theory of control over the plant's properties [10].

Here we also report two algorithms [8, 22-24] for synthesizing control systems for a nonlinear vector plant represented by a system of differential/difference equations, wherein the right-hand sides can contain unknown functions of two types of uncertainties: random or deterministic.

The basic tool for constructing all below-presented adaptive controllers, compensating for the bounded disturbances , is the method for Analytical Design of Aggregated discrete Regulators (ADAR) [1, 8] and the method of nonlinear adaptation on a manifold (NAD) (e.g., [1, 22]), whose importance is accounted for not only by the simplicity of the applied mathematical tool, to a great extent borrowed from the theoretical mechanics [25, 26] and its applications [27, 28], but also an account of the physical properties of an plant by including the given dynamic properties of the controlled plant into the target function. The proposed algorithms are derived analytically.

2. Formulation of the control problem with a deterministic disturbance

2.1. Formulation of a continuous-time control problem.

A nonlinear vector plant with description (1.1) is discussed. It is required to determine the law of control $\mathbf{u}(t) = \mathbf{U}\left(\mathbf{x}_1(t), \mathbf{x}_2(t)\right)$ in the state space for the sake of transferring the controlled plant (1.1) from a certain initial state $\mathbf{x}(0) = \left(\mathbf{x}_1^{\mathrm{T}}(0), \mathbf{x}_2^{\mathrm{T}}(0)\right)$ to the neighborhood of the manifold $\boldsymbol{\psi}(t) := \boldsymbol{\psi}\left(\mathbf{x}_1(t), \mathbf{x}_2(t)\right) = 0, t \to \infty$ and ensure an asymptotically stable motion of this system in this neighborhood. The following assumptions are currently available (see comments to the selection of a control quality functional in Appendix A):

1) The global minimum of the quality functional, including the least-action principle upon reaching the control target $\boldsymbol{\psi}\left(\mathbf{x}_1(t),\mathbf{x}_2(t)\right)=0, t\to\infty$, is given by the following:

$$\Phi_C=\int_0^{\infty}\sum_{l=1}^{3}\left(\psi_l^2(t)+w_l^2\dot{\psi}_l^2(t)\right)dt\to\min, \boldsymbol{\psi}(t)=0,\ t\to\infty,$$
$$\boldsymbol{\psi}=\left(\psi_1,\psi_2,\psi_3\right)^{\mathrm{T}}, \qquad (2.1)$$

where $\mathbf{w}=\left(w_1,w_2,w_3\right)^{\mathrm{T}}$ is the synthesized control system parameter regulating the time of motion of the imaging point of the system (2.1) to the neighborhood $\boldsymbol{\psi}\left(\mathbf{x}_1(t),\mathbf{x}_2(t)\right)=0, t\to\infty$; the subscript in the notation of the functional $\Phi_C\left(\Phi_D\right)$ indicates the continuity in time (discreteness in time) of the description of the controlled plant;

2) The boundedness of the right-hand side and the implementability of the system's behavior in the conditions of all admissible controls;

3) The availability of a stable target system for model (2.1), simultaneously responsible for dynamic and/or engineering requirements, not contradicting the physical capacity of the system [3, 4];

4) The disturbance, as a function of time, is such that the conditions for applying the ADAR-synthesis of plant control (2.1) hold.

Earlier [24], a number of statements on the conditions of availability of such a control were proved for a scalar plant.

2.2. Formulation of a discrete-time control problem with a deterministic disturbance

From the engineering viewpoint, it would make more physical sense [29, 30] to consider a discrete system obtained relying on (1.1), for instance, using the Euler scheme with a discretization parameter $\tau>0$, $t_d=d\cdot\tau$, $d=0,1,2,\ldots$

$$\mathbf{x}_1(d+1)=\mathbf{g}_1\left(\mathbf{x}_1(d),\mathbf{x}_2(d)\right),$$
$$\mathbf{x}_2(d+1)=\mathbf{g}_2\left(\mathbf{x}_1(d),\mathbf{x}_2(d)\right)+\zeta(d)+\mathbf{u}(d). \qquad (2.2)$$

Here systems (1.1) and (2.2) are related by the following expression:
$\mathbf{g}_i\left(\mathbf{x}_1(d),\mathbf{x}_2(d)\right)=\mathbf{x}_i(d)+\tau\cdot\mathbf{f}_i\left(\mathbf{x}_1(d),\mathbf{x}_2(d)\right),\ i=1,2.$

Remark 1. The basic assumption in the classical ADAR is the existence of an asymptotically stable system on a manifold with an attractive property assigned as a control target. From this perspective, it is possible to identify the conditions (in terms of probability) of a uniform proximity of the solutions $\mathbf{x}(t_d)$, $\mathbf{x}(d)$, $t_d=d\tau$, $d=0,1,2,\ldots$ $\mathbf{x}(t_d)$ to the systems (1.1) and (2.2), which was respectively obtained from (1.1) by the Euler time discretization [31]. However, the latter assumption requires a separate detailed and thorough analysis [31, 32] and is beyond the scope of this work.

Remark 2. An advantage of a discrete description is especially obvious for the plants with delayed control (see, e.g., p. 187 in [8]).

The formulation of the problem of discrete control formally differs from the continuous problem by the forms of the quality functional Φ_D and the equation, the solutions to which are represented by the stable extremals for functional Φ_D (see p. 172 in [8]):

$$\begin{aligned}
&\Phi_D = \sum_{d=0}^{\infty}\sum_{l=1}^{3}\left(\alpha_l^2\psi_l^2(d) + \left(\Delta\psi_l(d)\right)^2\right) \to \min,\\
&\boldsymbol{\psi}(d) = 0,\ d \to \infty,\ \boldsymbol{\psi} = \left(\psi_1, \psi_2, \psi_3\right)^{\mathrm{T}};\\
&\psi_l(d+1) + \omega_l\psi_l(d) = 0,\ \Delta\psi_l(d) = \psi_l(d+1) - \psi_l(d);\\
&\omega_l = 0.5\left(-\left(2+\alpha_l^2\right) \pm \sqrt{\left(2+\alpha_l^2\right)^2 - 4}\right),\ l = 1,2,3.
\end{aligned} \tag{2.3}$$

The relationship between the parameters α_l and ω_l in (2.3) immediately follows from the necessary condition of an extremum of the functional Φ_D – a discrete analog of the Euler–Lagrange equation.

Remark 3. Equations $w_l\dot{\psi}_l(t) + \psi_l(t) = 0,\ \psi_l(d+1) + \omega_l\psi_l(d) = 0,\quad l = 1,2,3;\ t,d \ge 0$ are the corollaries of the Euler–Lagrange equations for the respective functionals Φ_C and Φ_D.

3. Solution of the control problem on a manifold with a deterministic disturbance

There is a certain inconsistency in the 'adaptive control' terminology associated with the use of different adaptation principles (methods) (e.g., [9, 33-35]). In this study, integral adaptation on a manifolds understood as the process of variation of the averaged characteristics of a dynamic system with an uncertainty in the description of the operational environment, and an adaptation algorithm is the ability of asymptotically transfer the system's state into the neighborhood of a given invariant set in such an environment.

It is well known [8] that the synthesis of ADAR controls in the general case allows for a hierarchy of the control designing procedure. For instance, in the stabilization problems the control synthesis hierarchy is determined by a necessity to control the variables in the cases where a direct access to them is impossible. In so doing, a complex of partial criteria (target functionals) is used $\left\{\Phi_{C(D)}^{s}\right\}_{s=\overline{1,e}}$ with a certain subordination and consistency principle, each of the criteria represents a certain local requirement to the quality of the respective control sub-system. This approach to a certain extent agrees with the stability theory [36] and the idea of a nonreflexive cybernetic system [37].

3.1. Principle of nonlinear integral adaptation for deterministic plant disturbance suppression (NAD)

Step 1. Phase space extension by converting the external noise into an additional internal phase variable $\mathbf{z}$:

$$\begin{aligned}
&\dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2),\\
&\dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + \mathbf{z} + \mathbf{u},\\
&\dot{\mathbf{z}}(t) = \eta\boldsymbol{\psi}(t), \eta = \mathrm{const} > 0,
\end{aligned} \tag{3.1}$$

where the proportionality coefficient η is the synthesized control system parameter.

Step 2. Derivation of the synthesized controller structure by the classical ADAR synthesis methodology for a closed system (3.1). Note that at the final *e*-th level of the synthesis hierarchy the following form of the macrovariable is used: $\boldsymbol{\psi}^{(e)} = \boldsymbol{\psi} + k\mathbf{z},\ k = \text{const} > 0,$ where $\boldsymbol{\psi}$ is the target macrovariable from the control problem statement and the respective variational problem is formulated $\left(\Phi_C^e,\ \boldsymbol{\psi}^{(e)}\right)$

$$\Phi_C^e = \int_0^\infty \sum_{l=1}^{3} \left[\left(\psi_l^{(e)}\right)^2 + w_{e,l}^2 \left(\dot{\psi}_l^{(e)}\right)^2 \right] dt \to \min, \boldsymbol{\psi}^{(e)} \xrightarrow[t\to\infty]{} 0,$$
$$\boldsymbol{\psi}^{(e)} = \left(\psi_1^{(e)}, \psi_2^{(e)}, \psi_3^{(e)}\right)^{\mathrm{T}}, \tag{3.2}$$

with a linear functional control delivering a stable solution to Euler–Lagrange equation for the variation al problem $\left(\Phi_C^e,\ \boldsymbol{\psi}^{(e)}\right)$ (see Comment 2 in Appendix A)

$$\mathbf{w}_e \dot{\boldsymbol{\psi}}^{(e)}(t) + \boldsymbol{\psi}^{(e)}(t) = 0, t \to \infty. \tag{3.3}$$

Functional Φ_C^e is the quality criterion of the sought-for control during the motion of the imaging point of the system from the manifold $\boldsymbol{\psi}^{(e-1)}(t) = 0, t \to \infty$ to manifold $\boldsymbol{\psi}^{(e)}(t) = 0, t \to \infty.$ The superscript *e* (for system (3.1) *e*=1,2 depending on the type of the target macrovariable $\boldsymbol{\psi}$) denotes the final hierarchy number during the synthesis of a classical ADAR-regulator; the subscripts *e*, *l* used with the parameters $w_{e,l}$ indicate the hierarchy numbers and the vector coordinates, respectively; the multiplication $\mathbf{w}_e \dot{\boldsymbol{\psi}}^{(e)} = \left(w_{e,1}\dot{\psi}_1^{(e)}, w_{e,2}\dot{\psi}_2^{(e)}, w_{e,3}\dot{\psi}_3^{(e)}\right)^{\mathrm{T}}$ in (3.3) is coordinatewise.

All possible constants from (3.2), (3.3) $\mathbf{w}_s, s = \overline{1,2}$ are the control system parameters proportional to the velocity at which the imaging points reach the *s*-th subsystem of the neighborhood $\boldsymbol{\psi}_s(t) = 0, t \to \infty$ according to the partial criterion $\Phi_C^s, s = \overline{1,e}.$

The synthesis has been completed, and the final control system for model (1.1) represents a set of equations of the controlled plant (1.1) and the controller derived relying on (3.1).

Statement 1. Control $\mathbf{u} = \mathbf{u}_{NAD}$, if any, ensures an asymptotic stability for the controlled plant (1.1) in the neighborhood $\boldsymbol{\psi}(t) = 0, t \to \infty.$

The proof of Statement 1 is given in Appendix B.

3.2. Disturbance suppression algorithm based on discrete adaptation on a manifold for plant (2.2)

For the sake of definiteness, let us describe the algorithm for the stabilization problem with the control target

$$\boldsymbol{\psi}(d) = 0, d \to \infty,\ \boldsymbol{\psi}(d) = \mathbf{x}_1(d) - \mathbf{x}_1^*, \tag{3.4}$$

where $\mathbf{x}_1^*$ is the given target value for the variable $\mathbf{x}_1(d),\ d = 0,1,2,\dots,$ let $\omega_s, s = 1,2$ be scalar quantities.

Step 1. Extension of the model for system (2.2) given by

$$\begin{aligned}&\mathbf{x}_1(d+1)=\mathbf{g}_1(\mathbf{x}_1(d),\mathbf{x}_2(d)),\\&\mathbf{x}_2(d+1)=\mathbf{g}_2(\mathbf{x}_1(d),\mathbf{x}_2(d))+\mathbf{z}(d)+\mathbf{u}(d),\\&\mathbf{z}(d+1)=\mathbf{z}(d)+\gamma\boldsymbol{\psi}(d),\gamma=\text{const}>0.\end{aligned}\tag{3.5}$$

Step 2. Formulation and solution of the first variational problem $(\Phi_D^1,\psi^{(1)})$ for an plant with the description (3.4), (3.5):

$$J_D^1=\sum_{d=0}^{\infty}\sum_{l=1}^{3}\left(\alpha_{1,l}^2\left(\psi_l^{(1)}(d)\right)^2+\left(\Delta\psi_l^{(1)}(d)\right)^2\right)\to\min$$

with a restriction on $\boldsymbol{\psi}^{(1)}(d)=\mathbf{x}_2(d)-\boldsymbol{\varphi}(\mathbf{x}_1(d))=0, d\to\infty$, where $\boldsymbol{\varphi}(\mathbf{x}_1(d))$ is a further defined function.

From the equation, giving rise to stable extremals for Φ_D^1 $\boldsymbol{\psi}^{(1)}(d+1)+\omega_1\boldsymbol{\psi}^{(1)}(d)=0$, $|\omega_1|<1$, form the structure of the controller $\mathbf{u}(d)$, $d=1,2,..$

$$\begin{aligned}&\mathbf{u}(d)=-\mathbf{g}_2(d)-\mathbf{z}(d)+\breve{\boldsymbol{\varphi}}(d)-\omega_1\boldsymbol{\psi}^{(1)}(d),\ \mathbf{u}(d)=0,\\&\breve{\boldsymbol{\varphi}}(d):=\boldsymbol{\varphi}(\mathbf{g}_1(d)),\ d=1,2,..\end{aligned}\tag{3.6}$$

Step 3. Reduction (decomposition) on the manifold $\boldsymbol{\psi}^{(1)}(d)=0, d\to\infty$:

$$\begin{aligned}&\mathbf{x}_2(d)=\boldsymbol{\varphi}(\mathbf{x}_1(d)),\mathbf{x}_1(d+1)=\mathbf{g}_1(d)=\mathbf{g}_1(\mathbf{x}_1(d),\boldsymbol{\varphi}(\mathbf{x}_1(d))),\\&\mathbf{z}(d+1)=\mathbf{z}(d)+\gamma\boldsymbol{\psi}(d).\end{aligned}\tag{3.7}$$

Step 4. Formulation and solution of the second variational problem $(\Phi_D^2,\boldsymbol{\psi}^{(2)})$, $\boldsymbol{\psi}^{(2)}=\boldsymbol{\psi}+k\mathbf{z}$, $\boldsymbol{\psi}=\mathbf{x}_1-\mathbf{x}_1^*$, $k=\text{const}>0$ for an plant with the description (3.7):

$$J_D^2=\sum_{d=0}^{\infty}\sum_{l=1}^{3}\left(\alpha_{2,l}^2\left(\psi_l^{(2)}(d)\right)^2+\left(\Delta\psi_l^{(2)}(d)\right)^2\right)\to\min.$$

From equation $\boldsymbol{\psi}^{(2)}(d+1)+\omega_2\boldsymbol{\psi}^{(2)}(d)=0$, $|\omega_2|<1$, whose solutions are stable extremals for the functional Φ_D^2, form the function $\boldsymbol{\varphi}(\mathbf{x}_1(d))$ (assuming the fulfillment of all conditions for the existence of the ADAR controls):

$$\begin{aligned}&\mathbf{g}_1(\mathbf{x}_1(d),\varphi(\mathbf{x}_1(d)))-\mathbf{x}_1^*+k\gamma\boldsymbol{\psi}(d)+\\&+\omega_2(\boldsymbol{\psi}(d)+k(\mathbf{z}(d)+\gamma\boldsymbol{\psi}(d)))=0.\end{aligned}\tag{3.8}$$

Statement 2. Discrete control $\mathbf{u}$ (3.6), (3.8), if any, ensures an asymptotic stability for the controlled plant (2.2) in the neighborhood $\boldsymbol{\psi}(d)=0, d\to\infty$.

The proof of Statement 2 is similar to that of Statement 1, given the discreteness of description (Appendix B). The correctness of the controller structure (3.6), obtained relying on the extended system (3.5), follows from the limit relation $\boldsymbol{\psi}^{(2)}=\boldsymbol{\psi}+k\mathbf{z}=0, d\to\infty$. Substituting $\boldsymbol{\psi}(d)=-k\mathbf{z}(d)$ into the last equation in (3.5), we obtain $\mathbf{z}(d) \xrightarrow[d\to\infty]{} 0$, since $\mathbf{z}(d+1) = \mathbf{z}(d) + \gamma\boldsymbol{\psi}(d) = \mathbf{z}(d)(1-\gamma k), |1-\gamma k| < 1$, therefore, $\boldsymbol{\psi}(d) = -k\mathbf{z}(d) \xrightarrow[d\to\infty]{} 0.$

A summarizing description of the control system for the controlled plant (2.2) will, in this case, be given by a family of relations (2.2), (3.4), (3.6), and (3.8) with the parameters $\omega_1, \omega_2, \gamma, k,$ determining the transient process quality.

4. Formulation of a discrete problem of stochastic control on manifold

For random disturbances, the following representation [38] in the description of a discrete controlled plant will be convenient $(d = 0,1,...)$:

$$\begin{aligned} &\mathbf{x}_1(d+1) = \mathbf{g}_1(\mathbf{x}_1(d), \mathbf{x}_2(d)), \\ &\mathbf{x}_2(d+1) = \mathbf{g}_2(\mathbf{x}_1(d), \mathbf{x}_2(d)) + \boldsymbol{\xi}(d+1) + c\boldsymbol{\xi}(d) + \mathbf{u}(d). \end{aligned} \tag{4.1}$$

It is assumed that the disturbances are implemented as a sequence of independent equally distributed random quantities (functions of *d*) $\{\boldsymbol{\xi}(d)\}_{d\ge 0}, \boldsymbol{\xi}(d) = (\xi_1(d), \xi_2(d), \xi_3(d))$ with the following properties:

$$\begin{aligned} &\mathbf{E}\{\xi_j(d)\} = 0,\ \mathbf{E}\{\xi_i(d)\xi_j(d)\} = 0, \\ &\mathbf{D}\{\xi_j(d)\} = \sigma_j^2,\ i,j = 1,2,3,\ i \ne j. \end{aligned} \tag{4.2}$$

In (4.1), the constant $0 < c < 1$ is interpreted as the noise decay factor (the level of uncertainty in the subsequent point of time, *d*+1, is no smaller than in the current point of time *d*); in (4.2), $\mathbf{E}\{\cdot\}, \mathbf{D}\{\cdot\}$ are the operators of mathematical expectation and dispersion, respectively. It is required to determine the law of control $\mathbf{u}(d)$, ensuring the fulfillment of the following requirements:

$$\begin{aligned} &\mathbf{E}\{\psi_j(d+1) + \omega_j\psi_j(d)\} = 0, \\ &\mathbf{D}\{\psi_j(d+1) + \omega_j\psi_j(d)\} \to \min,\ j = 1,2,3,\ d \to \infty; \\ &\mathbf{E}\{\Phi_D\} = \mathbf{E}\left\{\sum_{d=0}^{\infty}\sum_{j=1}^{3}\left(\alpha_j^2(\psi_j(d))^2 + (\Delta\psi_j(d))^2\right)\right\} \to \min. \end{aligned} \tag{4.3}$$

The Nonlinear Adaptation on a manifold algorithm for a Stochastic plant (NAS) is an extension of the discrete version of the ADAR method including a stochastic plant, which was discussed in the earlier studies [23-24] for the 1-st–6-th order nonlinear descriptions as applied to biological (immunological) and economic systems. Let us briefly describe the main limitations and statements:

1) control strategies are selected from the class of discrete ADAR controls according to (2.3) and Section 3.2;

2) consider the strategies, for which the value of the control variable $\mathbf{u}(d)$ is a function of the current and previous states and controls $\mathbf{u}(d) = \mathbf{u}(\mathbf{x}(d), \mathbf{x}(d-1), ..., \mathbf{x}(0); \mathbf{u}(d-1), \mathbf{u}(d-2), ..., \mathbf{u}(0)),\ d = 0,1,...;$

3) select the strategies minimizing dispersion of the random variables – coordinates of the function for a fixed *d* - $\boldsymbol{\Psi}(d+1) + \boldsymbol{\omega}\boldsymbol{\Psi}(d)$, $|\omega_j| < 1, j = 1,2,3$, where $\boldsymbol{\Psi}(d)$ is the macrovariable in

the stage of determination of the controller structure $\mathbf{u}(d)$ (external control according to the terminology of the classical ADAR method [8]).

The main statements of the design of a discrete system of control over a nonlinear system (4.1) in the class of ADAR strategies minimizing dispersion of the coordinates of the target variable given by $\boldsymbol{\psi}(d) = \boldsymbol{\psi}(\mathbf{x}_1(d)) = 0,\ d \to \infty$ will be made up in the form of the NAS algorithm NAS [23].

NAS algorithm

Step 1. Searching for the structure of control $\tilde{\mathbf{u}}^A(d),\ d \in \{0,1,\ldots\}$ relying on the discrete version of ADAR under the conditions of fixed random disturbances.

Step 2. Taking the conditional expectation $\mathbf{u}(d) = \mathbf{E}\left\{\tilde{\mathbf{u}}^{\mathbf{A}}(d) \,\middle|\, \xi^d\right\}$, where $\xi^d = \left(\xi(0), \xi(1) \ldots \xi(d)\right)^{\mathrm{T}}$, $\xi(k) = \left(\xi_1(k), \xi_2(k), \xi_3(k)\right)^{\mathrm{T}}$, $k = \overline{1,d}$.

Step 3. Decomposing the description (4.1), taking into account the achieved limit relations $\boldsymbol{\psi}^{(1)}(d) = 0,\ d \to \infty$.

Step 4. Searching for the relationship $\xi(d) = \phi(\mathbf{x}(d), \boldsymbol{\psi}(d))$ as a function of the observation of an system whose behavior obeys the decomposed description (Step 3) in order to rule out the variable $\xi(d)$ from the expression $\hat{\mathbf{u}}(d) = \mathbf{E}\left\{\tilde{\mathbf{u}}^{\mathbf{A}}(d) \,\middle|\, \xi^d\right\}$. The synthesis of a stochastic discrete controller has been completed (see Appendix B).

Statement 3. Control $\mathbf{u} = \mathbf{u}_{NAS}$, if any, ensures a fulfillment of the requirements (4.3) and an asymptotic stability for the controlled plant (4.1), on average, in the neighborhood $\mathbf{E}\{\boldsymbol{\psi}(d)\} = 0,\ \boldsymbol{\psi}(d) = \mathbf{x}_1(d) - \mathbf{x}_1^*,\ d \to \infty$, where $\mathbf{x}_1^*$ is the given target value of the variable $\mathbf{x}_1(d)$.

The proof of Statement 3 is constructive and follows immediately from the NAS algorithms [18], the properties of the ADAR algorithm [8] and the selection of strategies minimizing dispersion of the output variable [38, 39]. By condition, a search for a stochastic control is performed from among the ADAR strategies based on the limit equalities given by $\psi_j^{(s)}(d+1) + \omega_{s,j}\psi_j^{(s)}(d) = 0,\ j = 1,2,3,\ s = 1,2$ at $d \to \infty$; note that the controller structure for system (4.1) is determined on the first hierarchy level. For any control, from the condition of independence of random functions (see Appendix B) $\xi(d), \xi(d+1), d = 0,1,\ldots$ one has an inequality (in a scalar notation)

$$\begin{aligned}
&\mathbf{D}\left\{\psi_j^{(1)}(d+1) + \omega_1\psi_j^{(1)}(d)\right\} = \mathbf{D}\left\{x_{2j}(d+1) - \varphi_j\left(x_{1j}(d+1)\right) + \omega_1\psi_j^{(1)}(d)\right\} = \\
&= \mathbf{D}\left\{g_{2j}(d) + u_j(d) - \breve{\varphi}_j\left(g_{1j}(d)\right) + \omega_1\psi_j^{(1)}(d) + c\xi_j(d)\right\} + \\
&+ \mathbf{D}\left\{\xi_j(d+1)\right\} \geq \sigma_j^2,\ j = 1,2,3.
\end{aligned} \tag{4.4}$$

From which follows that if a system is randomly disturbed, there is no control providing an error of the random function in the left-hand side of the functional equation with a dispersion smaller than the disturbance dispersion in the model (4.1) [23, 38]. A substitution of the found control into the system (4.1) would yield the limit equality

$$\boldsymbol{\psi}_j^{(1)}(d+1) + \omega_1\boldsymbol{\psi}_j^{(1)}(d) = \xi_j(d+1),\ d \to \infty,\ j = 1,2,3. \tag{4.5}$$

From relations (4.4) and (4.5) follows that the resulting control ensures the minimal dispersion $\mathbf{D}\left\{\psi_j^{(1)}(d+1) + \omega_1\psi_j^{(1)}(d)\right\} = \sigma_j^2,\ j = 1,2,3$ at $d \to \infty$, which makes Statement 3 correct.

5. Application of NAD and NAS algorithms in the problem of control over the center-of-mass motion of a disturbed moving plant

Use the plant's description from [15]:

$$\mathbf{M}\ddot{\mathbf{b}}(t) = \mathbf{q}\left(\mathbf{b}(t), \dot{\mathbf{b}}(t)\right) + \mathbf{u}(t), \ t \geq 0. \tag{5.1}$$

Here $\mathbf{b} \in R^3, \mathbf{b}(t) = \left(b_1(t), b_2(t), b_3(t)\right)^{\mathrm{T}}$ is the state vector with the coordinates; $\mathbf{q} = q\left(\mathbf{b}, \dot{\mathbf{b}}\right) \in R^3$ are the generalized forces acting on the controlled plant; $\mathbf{M} = \left\| m_{ij} \right\|_{3\times 3}$ is the kinetic energy matrix; $\mathbf{u} \in R^3$, $\mathbf{u} = \mathbf{u}\left(\mathbf{b}(t), \dot{\mathbf{b}}(t)\right)$ is the control vector.

It is required to design a control stabilizing the system's trajectories (1.1) $\mathbf{b}(t)$ in the neighborhood of the given values of $\mathbf{b}^*(t)$ from an arbitrary point of the state space $\mathbf{b}_0$ [10].

In order to implement the above-described NAD algorithm for synthesizing control, it is reasonable to transform system (5.1) to (1.1) substituting the variables $\mathbf{x}_1(t) = \mathbf{b}(t)$, $\mathbf{x}_2(t) = \dot{\mathbf{b}}(t)$:

$$\begin{aligned} &\dot{\mathbf{x}}_1(t) = \mathbf{x}_2(t), \\ &\dot{\mathbf{x}}_2(t) = \mathbf{A}\mathbf{q}(t) + \mathbf{A}\mathbf{u}(t) + \boldsymbol{\xi}(t), \ \mathbf{A} = \mathbf{M}^{-1}. \end{aligned} \tag{5.2}$$

Here we additionally introduce an uncertainty in the form of a limited continuous disturbance $\boldsymbol{\xi}(t)$. Thus, a nonlinear multidimensional system with a poorly formulated right-hand side in its description has been discussed.

5.1. Solution of the problem of nonlinear adaptation on a manifold for a plant (5.2) with a deterministic disturbance

In this case a problem is posed for finding control $\mathbf{u}(t) = \mathbf{u}(\mathbf{x}(t))$, $\mathbf{x}(t) = \mathbf{x} = \left(\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}}\right)^{\mathrm{T}}$ over an plant that would ensure its transfer from an arbitrary initial state $\mathbf{x}_0$ in a certain region of the space state into a given state $\mathbf{x}^*$ and its stabilization in a certain neighborhood of the target manifold

$$\boldsymbol{\psi}(\mathbf{x}) = \mathbf{x}_1 - \mathbf{x}^*, \tag{5.3}$$

where $\boldsymbol{\psi}(\mathbf{x})$ is the target macrovariable.

Remark 4. Derivation of the control system with variable $\mathbf{x}^*(t)$ is also quite straightforward but cumbersome, therefore, here for the sake of readability we take $\mathbf{x}^*(t)=\mathbf{x}^*=$const, and parameters w_1, w_2 are also taken to be scalar.

Implementation of the NAD algorithm

Step 1. Extension of the system:

$$\begin{aligned} &\dot{\mathbf{x}}_1(t) = \mathbf{x}_2(t), \\ &\dot{\mathbf{x}}_2(t) = \mathbf{A}\mathbf{q}(t) + \mathbf{A}\mathbf{u}(t) + \mathbf{z}(t), \\ &\dot{\mathbf{z}}(t) = \eta\boldsymbol{\psi}(t), \eta = \mathrm{const} > 0. \end{aligned} \tag{5.4}$$

Step 2. Derivation of the controller structure.

2.1. Introduce the macrovariable $\boldsymbol{\psi}^{(1)} = \mathbf{x}_2 - \boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z})$, in designing which a further defined function is used $\boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z})$.

2.2. Formulate the variational problem $\left(\Phi_C^1,\ \boldsymbol{\psi}^{(1)}\right)$, where $\Phi_C^1 = \int_0^\infty \sum_{l=1}^{3} \left[\left(\psi_l^{(1)}\right)^2 + w_1^2\left(\dot{\psi}_l^{(1)}\right)^2\right] dt \to \min$, and the macrovariable $\boldsymbol{\psi}^{(1)} = \left(\psi_1^{(1)}, \psi_2^{(1)}, \psi_3^{(1)}\right)^{\mathrm{T}}$ determines the limit given by $\boldsymbol{\psi}^{(1)}(t) = 0, t \to \infty$. The problem solution $\left(\Phi_C^1,\ \boldsymbol{\psi}^{(1)}\right)$ relies on the equation $w_1 \dot{\boldsymbol{\psi}}^{(1)}(t) + \boldsymbol{\psi}^{(1)}(t) = 0,\ w_1 = \mathrm{const} > 0.$

2.3. Control $\mathbf{u}$ is determined by the following relations:

$$\mathbf{A}\mathbf{u} = -w_1^{-1}\boldsymbol{\psi}^{(1)} - \mathbf{A}\mathbf{q} + \mathbf{J}(\boldsymbol{\varphi})\dot{\mathbf{Y}}^{\mathrm{T}} - \mathbf{z}(t),$$
$$\dot{\mathbf{Y}} = \begin{bmatrix} \dot{x}_{11} & \dot{z}_1 \\ \dot{x}_{12} & \dot{z}_2 \\ \dot{x}_{13} & \dot{z}_3 \end{bmatrix} = \begin{bmatrix} x_{21} & \eta\psi_1 \\ x_{22} & \eta\psi_2 \\ x_{23} & \eta\psi_3 \end{bmatrix},\ \mathbf{J}(\boldsymbol{\varphi}) = \begin{bmatrix} \dfrac{\partial\varphi_1}{\partial x_{11}} & \dfrac{\partial\varphi_2}{\partial x_{12}} & \dfrac{\partial\varphi_3}{\partial x_{13}} \\ \dfrac{\partial\varphi_1}{\partial z_1} & \dfrac{\partial\varphi_2}{\partial z_2} & \dfrac{\partial\varphi_3}{\partial z_3} \end{bmatrix}^{\mathrm{T}}. \tag{5.5}$$

2.4. System (5.4) is reduced on the attained manifold $\boldsymbol{\psi}^{(1)} = 0$, where one has the relation $\mathbf{x}_2 = \boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z}), t \to \infty$:

$$\begin{aligned} \dot{\mathbf{x}}_1(t) &= \boldsymbol{\varphi} = \boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z}), \\ \dot{\mathbf{z}}(t) &= \eta\boldsymbol{\psi} = \eta\left(\mathbf{x}_1(t) - \mathbf{x}^*\right). \end{aligned} \tag{5.6}$$

System (5.6) describes the law of behavior of (5.4) on the manifold $\mathbf{x}_2 = \boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z})$.

2.5. In order to define the function $\boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z})$ introduce a macrovariable $\boldsymbol{\psi}^{(2)} = \boldsymbol{\psi} + k\mathbf{z},\ k = \mathrm{const} > 0$ and pose a variational problem $\left(\Phi_C^2,\ \boldsymbol{\psi}^{(2)}\right)$: $\Phi_C^2 = \int_0^\infty \sum_{l=1}^{3} \left[\left(\psi_l^{(2)}\right)^2 + w_2^2\left(\dot{\psi}_l^{(2)}\right)^2\right] dt \to \min$ in accordance with which the control $\boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z})$ for the plant (5.6) is determined from the equation $w_2 \dot{\boldsymbol{\psi}}^{(2)}(t) + \boldsymbol{\psi}^{(2)}(t) = 0$:

$$\boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z}) = -w_2^{-1}\boldsymbol{\psi}^{(2)} - k\eta\boldsymbol{\psi} = -w_2^{-1}(\boldsymbol{\psi} + k\mathbf{z}) - k\eta\boldsymbol{\psi}. \tag{5.7}$$

Here Φ_C^2 is the quality functional of the sought-for control during the motion of the imaging point from the manifold $\boldsymbol{\psi}^{(1)} = 0$ to the manifold $\boldsymbol{\psi}^{(2)} = 0$.

The synthesis is over, and the final control system for the model (5.2) represents a family of equations of the controlled plant (5.2) and the controller (5.5), (5.7) with the parameters w_1, w_2, k, η in the scalar notation given by the following:

$$\begin{aligned}
&(\mathbf{Au})_i = -w_1^{-1}\psi_i^{(1)} - (\mathbf{Aq})_i + \frac{\partial\varphi_i}{\partial x_{1i}} x_{2i} + \eta\frac{\partial\varphi_i}{\partial z_i}\psi_i - z_i,\\
&\dot z_i(t) = \eta\psi_i, \psi_i = x_{1i}(t) - x_i^*, \eta > 0,\\
&\varphi_i = -w_2^{-1}\left(\psi_i + kz_i(t)\right) - k\eta\psi_i, \frac{\partial\varphi_i}{\partial x_{1i}} = -\left(w_2^{-1} + k\eta\right), \frac{\partial\varphi_i}{\partial z_i} = -w_2^{-1}k,\\
&\psi_i^{(1)} = x_{2i} - \varphi_i\left(x_{1i}, z_i\right), \psi_i^{(2)} = \psi_i(x) + kz_i, k > 0, i = 1,2,3.
\end{aligned} \tag{5.8}$$

The control system (5.2), (5.8) suppresses the external disturbances without any preliminary estimation of the system's coordinates (Figs. 1 and 2) and provides the asymptotically stable target properties for the controlled plant (5.2).

The following are the main positive properties of the above-described NAD algorithm for designing control for the plant (5.2):

1) no information is required on the character of the uncertainty ξ but its boundedness and the fulfillment of natural conditions of implementability of the ADAR algorithm;
2) control is continuous;
3) all positive control properties are inherited from the ADAR methods and from the sliding-mode control: asymptotically stable attainment of the target manifold; control robustness in the sliding mode with respect to the uncertainties;
4) least-action principle with a given Lagrangian is implemented, which ensures that the control target is reached by the plant with a given velocity due to the given controller parameters $w_1, w_2 > 0$ – the coefficients at the derivatives $\dot{\boldsymbol{\psi}}^{(1)}, \dot{\boldsymbol{\psi}}^{(2)}$.

A troublesome feature of this approach is the assumption of a disturbance in terms of the controllable variables only.

5.2. Results of numerical simulation of the NAD control

The designed algorithm has been implemented for the model technological task [15, 40]: testing to which extent the designed implement can execute precise positioning into a given point of the working space (Fig. 1) under the conditions of continuous disturbances $\xi = (0.2;\ 0.2;\ 0.3)^{\mathrm{T}}$; $\mathbf{x}_1(0) = (\pi/8;\ 0;\ 0)^{\mathrm{T}}$, parameter values $w_i = 0.4,\ i = 1,2$ and (Fig. 2) – under the conditions of harmonic disturbances $\xi_i(t) = 0.3\sin(\pi t/n),\ i = 1,2,3,\ t = 0,1,\ldots;\ \mathbf{x}_1(0) = (1.2;\ 1;\ 0)^{\mathrm{T}}$, parameter values $w_1 = 0.9;\ w_2 = 0.09;\ \eta = k = 0.1$.

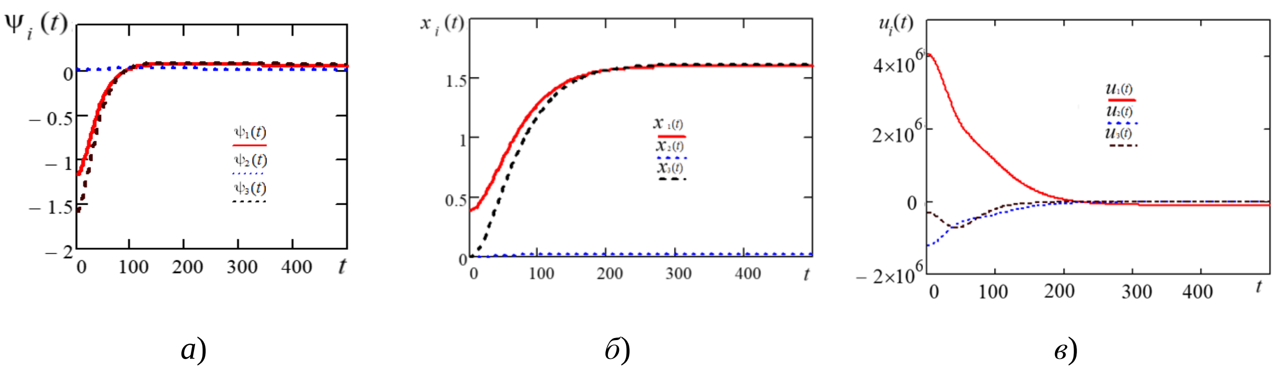
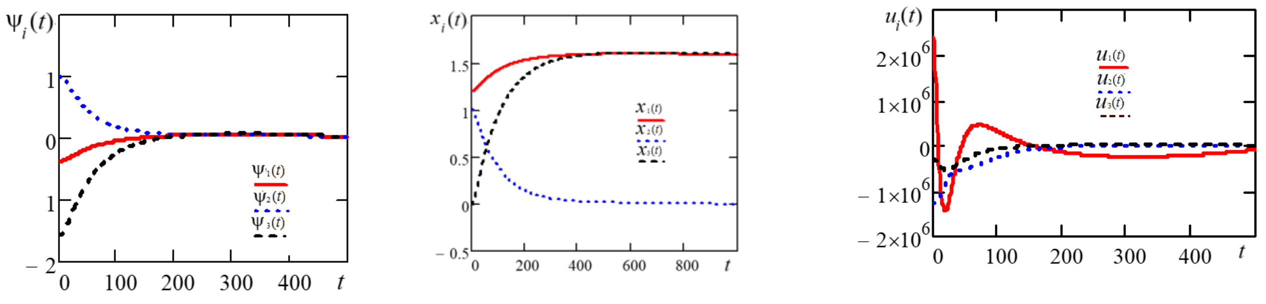


Fig. 1. *a*) - behavior of target macrovariable, *b*) - phase vector coordinates, and *c*) - controls of system (5.8) under the conditions of continuous disturbances



а) *б*) *в*)

Fig. 2. *a*) - target macrovariable, *b*) - phase vector coordinates and *c*) - controls of system (5.8) under the conditions of harmonic disturbances.

Modeling of a control system as a collection of equations (5.2), (5.3), (5.8) was carried out for the following input data values:

Initial state – vector $\mathbf{x}_1(0)=(\pi/8;\ 0;\ 0)^{\mathrm{T}}$ (rad.). Target values $\mathbf{x}^*(0)=\left(1.571;\ -8.308\times10^{-7};\ 1.571\right)^{\mathrm{T}}$.

Control system parameters $w_i>0, i=1,2$ are the quantities directly proportional to the duration of transient processes.

The control variables (hinge moments) acquire the values from the intervals: $u_1=M_1\in\left[-64\cdot10^6,\ 64\cdot10^6\right]$, $u_i=M_i\in\left[-32\cdot10^6,\ 32\cdot10^6\right], i=2,3$ (N/m) [40].

5.3. Application of the NAS algorithm

According to NAS, the final description [41] of the discrete stochastic control system with the parameters ω_1,ω_2,τ is made up by the following collection of equations:

$$\begin{aligned}
&\mathbf{x}_1(d+1)=\mathbf{g}_1(d),\ \mathbf{g}_1(d)=\mathbf{x}_1(d)+\tau\mathbf{x}_2(d),\\
&\mathbf{x}_2(d+1)=\mathbf{g}_2(d)+\tau\mathbf{A}\mathbf{u}(d)+\xi(d+1)+c\xi(d),\\
&\mathbf{g}_2(d)=\mathbf{x}_2(d)+\tau\mathbf{A}\mathbf{q}\left(x_1(d),\mathbf{x}_2(d)\right),\\
&\boldsymbol{\psi}\left(\mathbf{x}_1(d)\right)=\mathbf{x}_1(d)-\mathbf{x}^*,\ \boldsymbol{\psi}^{(1)}(d)=\mathbf{x}_2(d)-\boldsymbol{\varphi}\left(\mathbf{x}_1(d)\right),\\
&\tau\mathbf{A}\mathbf{u}(d)=-\mathbf{g}_2(d)-c\omega_1\boldsymbol{\psi}^{(1)}(d-1)-(1+\omega_1+c)\boldsymbol{\psi}^{(1)}(d)-\omega_2\mathbf{x}_2(d),\\
&\boldsymbol{\varphi}\left(\mathbf{x}_1(d)\right)=-\tau^{-1}(1+\omega_2)\boldsymbol{\psi}\left(\mathbf{x}_1(d)\right), d\in\{0,1,\ldots\}.
\end{aligned}\tag{5.9}$$

Statement 4. The estimates of disturbances as a function of the observed variables, obtained via the NAD (NAS) algorithms, delivering to the control systems an asymptotic stability in the neighborhood of the target manifold (on average) are respectively given by the following:

$$\begin{aligned}
&\dot{\mathbf{z}}(t)=\eta\boldsymbol{\psi}(t),\eta>0;\\
&\hat{\xi}(d)=\boldsymbol{\psi}^{(1)}(d)+\omega_1\boldsymbol{\psi}^{(1)}(d-1),|\omega_1|<1.
\end{aligned}\tag{5.10}$$

The proof of Statement 4 is constructive and follows immediately from the NAD and NAS algorithms. The validation of the first equation from (5.10) lies in the proof of Statement 1, and the correctness of the second equation follows from the proof of Statement 3.

From (5.10) follows that the model for estimating deterministic disturbances (the last equation in the system (5.4)), delivering an asymptotic stability to the control system in the neighborhood of the target manifold, has a well-defined validation.

6. Comparison of the deterministic and stochastic controls in the design and off-design conditions

Consider the results of modeling the control system (5.9) (Fig. 4) for the above-enumerated parameter values, initial conditions with the target vector $\mathbf{x}_1^* = \left(1.571;\ -8\cdot 10^{-7};\ 1.571\right)^{\mathrm{T}}$ and the discretization step of $\tau = 10^{-5}$ arbitrary units.

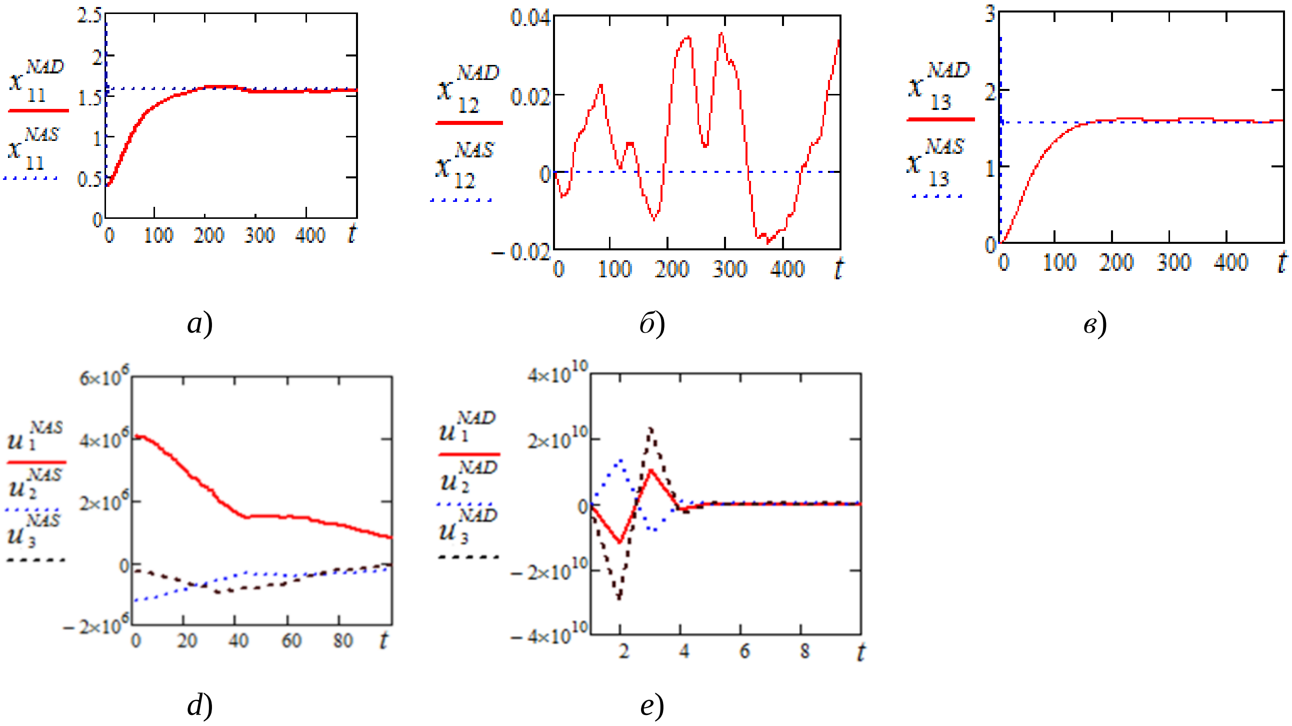


Fig. 3. *a*), *b*), *c*) – comparative graphs of the controlled variables and *d*), *e*) – controls of the systems (5.8) and (5.9) under the conditions of random disturbances $\xi_i \sim N(0;1)$.

As it follows from Figs. 3 *d*), *e*), the NAD control (for the sake of simulation we discretized the systems (5.2), (5.8) using the Euler scheme) is even more efficient in terms of energy in the off-design conditions, but Figs. 3 *a*), *b*), *c*) suggest a higher precision and stability of the NAS control in the cases where the target values are attained. A more careful numerical study should be carried out by selecting the set of values of w_1, w_2, τ, η, k $(\omega_1, \omega_2, \tau)$ substantially affecting the transient process quality. It is also clear that there is an overshoot problem in the discrete NAS algorithm, which is still an open issue.

Summing up, the method of nonlinear adaptation on the manifold is applicable both to continuous and discrete systems both under the conditions of random and deterministic disturbances. However, a limitation of the algorithm of nonlinear adaptation on a manifold proposed here is the presence of disturbances only in the equations containing the control variable, while in the stochastic algorithm this limitation is eliminated.

7. Conclusions

The two reported algorithms for the solution of the problem of control over the center of mass motion of a moving plant [13-15] with an incomplete description represent a continuation of the previous research [22-24].

It should be noted that both algorithms admit a control system design with a restriction of the values of the variables and control, but the design of a control meeting such additional requirements is yet the focus of future research.

We have presented the results of numerical simulations validating the performance of the designed controllers in the class of the ADAR controls for the plants with random and deterministic uncertainties in the description.

These results can be relevant in, e.g., all control systems of robotic arms, functioning under the conditions of uncertainty, and in designing the decision-making support systems in controlling compound plants.

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APPENDIX

A. Commentary to the variational problem $\left(\Phi_C,\ \psi\right)$.

Comment 1. The results of theoretical mechanics (see e.g., [26], and [8], pp. 77-82) imply the validity of the following statement: for the relation $\psi=\psi\left(x_1,x_2\right)=0$ (here for simplicity the dimensionality of the plant *n*=2 and the control is scalar) to be an invariant manifold of the system $\dot{x}_i=R_i\left(x_1,x_2\right),\ i=1,2$, it is necessary and sufficient that the following condition be fulfilled:

$$\frac{d\psi}{dt}=\sum_{i=1}^{2}\frac{\partial\psi\left(x_1,x_2\right)}{\partial x_i}R_i\left(x_1,x_2\right)=\phi\left(\psi,x_1,x_2\right),$$
$$R_i\left(x_1,x_2\right)=f_i\left(x_1,x_2\right)+u_{1(2)},$$

where $u_{1(2)}$ enters the first or the second equation; $\phi\left(\psi,x_1,x_2\right)$ is a certain single-valued (with respect to ψ) continuous differentiable function of its arguments, which would turn to zero at $\psi=0:\phi\left(0,x_1,x_2\right)=0$, specifically $\phi\left(\psi,x_1,x_2\right)=w\psi\left(x_1,x_2\right),\ \ w=\text{const},\ w>0$.

Comment 2. If the variational problem $\left(\Phi_C,\ \psi\right)$ is given by

$$\Phi_C=\int_0^{\infty}\left(\phi^2\left(\psi\right)+w^2\dot{\psi}^2\right)dt\rightarrow\min,\psi=0,\ t\rightarrow\infty,$$
$$\phi\left(\psi\right)\psi>0,\ \forall\psi\neq0,\ \phi\left(0\right)=0,$$

then the solutions of the functional equation $w\dot{\psi}+\phi\left(\psi\right)=0$ would determine the stable extremals delivering a minimum to the functional Φ_C.

Indeed, the Euler–Lagrange equation for Φ_C with the subintegral function $F\left(t,\psi,\dot{\psi}\right)=\phi^2\left(\psi\right)+w^2\dot{\psi}^2$ will be given by: $\frac{\partial F}{\partial\psi}-\frac{d}{dt}\frac{\partial F}{\partial\dot{\psi}}=2\phi\left(\psi\right)\frac{\partial\phi}{\partial\psi}-2w^2\ddot{\psi}=0$. Multiplying the latter equation by $\dot{\psi}$ and integrating with respect to *t*, obtain

$$\phi\left(\psi\right)\frac{\partial\phi}{\partial\psi}\dot{\psi}-w^2\ddot{\psi}\dot{\psi}=0\Rightarrow\int\phi\left(\psi\right)d\phi-w^2\int\dot{\psi}d\dot{\psi}=0\Rightarrow\phi^2\left(\psi\right)-w^2\dot{\psi}^2=C,$$

wherefrom, considering the conditions $\psi=0,\ \dot{\psi}=0,\ \phi\left(0\right)=0,\ t\rightarrow\infty$, constant $C=0$. Rewriting the resulting expression in the form of $\left(\phi\left(\psi\right)-w\dot{\psi}\right)\left(\phi\left(\psi\right)+w\dot{\psi}\right)=0$, we observe that the solutions to the equation $w\dot{\psi}+\phi\left(\psi\right)=0$ possess the properties of the asymptotic stability.

B. Proof of Statement 1. By condition, disturbance $\xi(t)$ in the description (1.1) is a limited, continuous function of time, and its behavior does not involve any violation of the conditions of the existence and performance of the ADAR control (Section 2.1).

It is well known that *a*) continuous functions (here disturbances) may be described as the solutions to certain differential equations [42]; *b*) continuous disturbances may be represented by a superposition of piecewise-continuous (piecewise-constant, in particular) basis functions with the coefficients changing in a stepwise manner within comparatively short time periods (wave representation [28]); *c*) any measurable bounded function may be represented as a uniform limit of piecewise-constant functions (see e.g., [43]). In view of the above, it is suffice to demonstrate the attainability of the given target manifold and the system's stabilization in its neighborhood for such disturbances as piecewise-constant functions of time [11].

Under the conditions of implementability of the NAD algorithm (and ADAR method, respectively (Items 1) - 4) in Section 2.1), the initial system for the regulator synthesis is given by (2.1) (Step 1 of the NAD algorithm, Section 3.1). Now consider possible versions of the form of the target macrovariable $\boldsymbol{\psi}$ for the plant (1.1): $\boldsymbol{\psi}=\boldsymbol{\psi}(\mathbf{x}_1,\mathbf{x}_2), \boldsymbol{\psi}=\boldsymbol{\psi}(\mathbf{x}_1)$. Case $\boldsymbol{\psi}=\boldsymbol{\psi}(\mathbf{x}_2)$ is trivial and represents a special case of the former, since the control is contained in the second equation.

1. Consider a problem of control over the plant (1.1) with the macrovariable $\boldsymbol{\psi}=\boldsymbol{\psi}(\mathbf{x}_1,\mathbf{x}_2)$. In the description of $\boldsymbol{\psi}=\boldsymbol{\psi}(\mathbf{x}_1,\mathbf{x}_2)$, there are both state variables in place of arguments, therefore the invariant $\boldsymbol{\psi}(\mathbf{x}_1,\mathbf{x}_2)=0$ is attainable within a single step of the NAD algorithm, according to which we take $\boldsymbol{\psi}^{(1)}=\boldsymbol{\psi}+k\mathbf{z},\ k>0$ and, given the closed system (2.1), obtain

$$w_1\dot{\boldsymbol{\psi}}^{(1)}+\boldsymbol{\psi}^{(1)}=w_1(\dot{\boldsymbol{\psi}}+k\eta\boldsymbol{\psi})+\boldsymbol{\psi}+k\mathbf{z}=0, t\to\infty,$$

$$\dot{\boldsymbol{\psi}}=\mathbf{J}(\boldsymbol{\psi})\cdot\mathbf{F}^T, \mathbf{J}(\boldsymbol{\psi})=\begin{bmatrix}\frac{\partial\psi_1}{\partial x_{11}} & \frac{\partial\psi_2}{\partial x_{12}} & \frac{\partial\psi_3}{\partial x_{13}}\\ \frac{\partial\psi_1}{\partial x_{21}} & \frac{\partial\psi_2}{\partial x_{22}} & \frac{\partial\psi_3}{\partial x_{23}}\end{bmatrix}^{\mathrm{T}}, \mathbf{F}=\begin{bmatrix} f_{11} & f_{21}+z_1+u_1\\ f_{12} & f_{22}+z_2+u_2\\ f_{13} & f_{23}+z_3+u_3\end{bmatrix},$$

wherefrom there follows the law of control $\mathbf{u}=(u_1,u_2,u_3)^{\mathrm{T}}$, providing $\boldsymbol{\psi}^{(1)}=\boldsymbol{\psi}+k\mathbf{z}=0,\ t\to\infty$:

$$\begin{gathered} u_i=-\left(\frac{\partial\psi_i}{\partial x_{2i}}\right)^{-1}\left(w_1^{-1}(\psi_i+kz_i)+k\eta\psi_i+\frac{\partial\psi_i}{\partial x_{1i}}f_{1i}\right)-f_{2i}-z_i,\\ \frac{\partial\psi_i}{\partial x_{2i}}\neq 0,\ z_i=\eta\int\psi_i dt,\ i=1,2,3. \end{gathered} \tag{A.1}$$

The final control system represents a collection of the plant's (1.1) and regulator's (A.1) equations, whose structure has an integrator providing a suppression of the piecewise-constant disturbance additively entering the equation with control (a system with the 1-st order astaticism with respect to the master control) in the steady-state mode of the system operation.

It is easy to show that after having taken the Lyapunov's function in the form of $V=0.5\sum_{i=1}^{3}\left(\psi_i^{(1)}\right)^2$, its derivative $\dot{V}$ will be negative $\dot{V}=-w_1^{-1}\sum_{i=1}^{3}\left(\psi_i^{(1)}\right)^2<0$ upon substitution of (A.1) taking into account equations (2.1). Substituting the control (A.1) into the description (1.1), obtain

$$\dot{\boldsymbol{\psi}}^{(1)}+w_1^{-1}\boldsymbol{\psi}^{(1)}=\mathrm{diag}\left(\frac{\partial\psi_1}{\partial x_{21}},\frac{\partial\psi_2}{\partial x_{22}},\frac{\partial\psi_3}{\partial x_{23}}\right)(\xi-\mathbf{z})^{\mathrm{T}}, \tag{A.2}$$

or, in the scalar description, $w_1\dot{\psi}_i^{(1)}+\psi_i^{(1)}=w_1\frac{\partial\psi_i}{\partial x_{2i}}(\xi_i-z_i),\ i=1,2,3.$

2. Consider now the problem of control over the plant (1.1) with the macrovariable $\boldsymbol{\psi}=\boldsymbol{\psi}(\mathbf{x}_1)$. Attaining the invariant $\boldsymbol{\psi}(\mathbf{x}_1)=0$ is provided by the NAD algorithm in two steps. First, from the solution of the functional equation $w_1\dot{\boldsymbol{\psi}}^{(1)}(t)+\boldsymbol{\psi}^{(1)}(t)=0, t\to\infty$ for the problem $(\Phi_C^1,\boldsymbol{\psi}^{(1)}), \boldsymbol{\psi}^{(1)}=\mathbf{x}_2-\boldsymbol{\varphi}(\mathbf{x}_1,\mathbf{z})$ we obtain, considering the equations of the extended system,

$$\mathbf{u} = -w_1^{-1}\boldsymbol{\psi}^{(1)} + \dot{\boldsymbol{\varphi}}(\mathbf{x}_1, \mathbf{z}) - \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) - \mathbf{z}, \mathbf{z} = \eta\int \boldsymbol{\psi} dt \tag{A.3}$$

Decomposition of the system (2.1) on the manifold $\mathbf{x}_2 = \boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z}), t \to \infty$ will yield the next system, which would be an input for the second stage of synthesis

$$\begin{aligned} &\dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z})), \\ &\dot{\mathbf{z}} = \eta\boldsymbol{\psi}, \eta > 0. \end{aligned} \tag{A.4}$$

Find the expressions in order to define $\boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z})$ from the solution of the functional equation $w_2\dot{\boldsymbol{\psi}}^{(2)}(t) + \boldsymbol{\psi}^{(2)}(t) = 0, t \to \infty$ for the problem $(\Phi_C^2, \boldsymbol{\psi}^{(2)}), \boldsymbol{\psi}^{(2)} = \boldsymbol{\psi} + k\mathbf{z},\ k > 0$

$$\begin{aligned} &w_2(\dot{\boldsymbol{\psi}} + k\eta\boldsymbol{\psi}) + \boldsymbol{\psi} + k\mathbf{z} = 0, \\ &\dot{\boldsymbol{\psi}} = \begin{bmatrix} \dfrac{d\psi_1}{dx_{11}} f_{11} & \dfrac{d\psi_2}{dx_{12}} f_{12} & \dfrac{d\psi_3}{dx_{13}} f_{13} \end{bmatrix}^{\mathrm{T}}, \\ &\mathbf{f}_1 = (f_{11}, f_{12}, f_{13})^{\mathrm{T}}, \mathbf{f}_1 = \mathbf{f}_1(\mathbf{x}_1, \boldsymbol{\varphi}(\mathbf{x}_1, \mathbf{z})), \end{aligned} \tag{A.5}$$

so further we can also define its total derivative. The regulator for the plant (1.1) with the control target $\boldsymbol{\psi}(t) = \boldsymbol{\psi}(\mathbf{x}_1(t)) = 0, t \to \infty$ is determined by a collection of equations (A.3)-(A.5). Substituting the resulting control (A.3) into the description (1.1), obtain

$$\begin{aligned} &\dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + \boldsymbol{\xi} - w_1^{-1}\boldsymbol{\psi}^{(1)} + \dot{\boldsymbol{\varphi}}(\mathbf{x}_1, \mathbf{z}) - \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) - \mathbf{z} \Rightarrow \\ &\dot{\mathbf{x}}_2 + w_1^{-1}\boldsymbol{\psi}^{(1)} - \dot{\boldsymbol{\varphi}}(\mathbf{x}_1, \hat{\mathbf{z}}) = \boldsymbol{\xi} - \mathbf{z} \Rightarrow w_1\dot{\boldsymbol{\psi}}^{(1)} + \boldsymbol{\psi}^{(1)} = w_1(\boldsymbol{\xi} - \mathbf{z}). \end{aligned} \tag{A.6}$$

The designed controls (A.1), (A.3) have been obtained from the condition $w_1\dot{\boldsymbol{\psi}}^{(1)}(t) + \boldsymbol{\psi}^{(1)}(t) = 0, t \to \infty$ in the space of the plant's states (1.1), ensures that for ε>0 $|\boldsymbol{\xi}(t) - \mathbf{z}(t)| < \varepsilon$, starting from a certain value of t_ε considering (A.2) and (A.6). Statement 1 has been proved.

B. Commentary to the NAS algorithm. Step 1 of the NAS algorithm, based on the classical ADAR, would yield an intermediate relation (given the functions $\xi(d), \xi(d+1)$ are fixed)

$$\begin{aligned} &\tilde{\mathbf{u}}^A(d) = -\mathbf{g}_2(d) - (\xi(d+1) + c\xi(d)) + \breve{\boldsymbol{\varphi}}(d) - \omega_1\boldsymbol{\psi}^{(1)}(d), \\ &\breve{\boldsymbol{\varphi}}(d) := \boldsymbol{\varphi}(\mathbf{g}_1(d)),\ d = 1, 2, .. \end{aligned}$$

The fulfillment of the operation of conventional mathematical expectation $\hat{\mathbf{u}}(d) = \mathbf{E}\{\tilde{\mathbf{u}}^{\mathbf{A}}(d) \,|\, \xi^d\}$ in Step 2 of the NAS algorithm would yield the following form:

$$\hat{\mathbf{u}}(d) = -\mathbf{g}_2(d) - c\xi(d) + \breve{\boldsymbol{\varphi}}(d) - \omega_1\boldsymbol{\psi}^{(1)}(d),\ d = 1, 2, .. \tag{A.7}$$

By decomposing (Step 3) the description (3.1) including (A.7) we obtain the relation

$$\begin{aligned} &\boldsymbol{\psi}^{(1)}(d+1) + \omega_1\boldsymbol{\psi}^{(1)}(d) = \mathbf{x}_2(d+1) - \breve{\boldsymbol{\varphi}}(d) + \omega_1\boldsymbol{\psi}^{(1)}(d) = \\ &= \mathbf{g}_2(d) + \mathbf{u}(d) - \breve{\boldsymbol{\varphi}}(d) + \omega_1\boldsymbol{\psi}^{(1)}(d) + c\xi(d) + \xi(d+1) = \xi(d+1), \end{aligned} \tag{A.8}$$

where from follows that an estimation of the disturbance as a function of state, delivering the minimal dispersion to the random function coordinates $\boldsymbol{\psi}^{(1)}(d+1)+\omega_1\boldsymbol{\psi}^{(1)}(d)$, will be given by

$$\hat{\boldsymbol{\xi}}(d)=\boldsymbol{\psi}^{(1)}(d)+\omega_1\boldsymbol{\psi}^{(1)}(d-1). \qquad \text{(A.9)}$$

Substituting (A.9) into (A.7), we obtain the final expression for control

$$\mathbf{u}(d)=-\mathbf{g}_2(d)-c\left(\boldsymbol{\psi}^{(1)}(d)+\omega_1\boldsymbol{\psi}^{(1)}(d-1)\right)+\breve{\boldsymbol{\varphi}}(d)-\omega_1\boldsymbol{\psi}^{(1)}(d),\ d=1,2,..$$